VIBRATION AND BUCKLING OF GENERALLY ORTHOTROPIC PLATES

A thesis submitted in partial fulfilment of the Requirements for the Degree of MASTER OF TECHNOLOGY



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CERTIFIED that this work has been carried out under my supervision and that it has not been submitted else-where for a Degree.

(D. V. SUNDAMMAN)

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NOTATION

X, y, z	Rectangular coordinates
	length of plate along maxis
b	Width of plate along y-axis
h	Thickness of plate along s-axis
u,v,w	Components of displacements along x, y and z directions.
81, S2, S3	Strains along x,y, and z directions.
Tq (q = 1,2 6)	Normal and shear stresses defined in equation A-3.
bqr(q,re 1,2 6)	Elastic stiffness of generally orthotropic plate defined by equation A-2.
M ₁ , M ₂ , M ₁₂ , M ₂₁	Bending and Twisting moments per unit length.
13, 18	implane loads parallel to x and y directions per unit length.
Q1. Q2	Plate shears
Daj (1,je 1,26)	Florural and twisting rigidities defined by equation 2.7
8	Mass per unit area of the plate . ph
f	Poissons ratio
y	Inertia force acting on unit area of plate
	A
•	Circular frequency
	Time
Cent	Goefficients in series expansion of plate deflection
•	Angle of orthotropicity
	Total potential energy of plate

Non-dimensional inclane load Ap Ap Ap coefficients D126 = D18 + E D66 Mon-dimensional frequency parameter (W 7 m/ 1 D;) 1/2 B Side ratio (a/b) I Unitary metrix -Column metrix of coefficients Com Beam characteristic function Constants in boom characteristic Ber Le functions 2 Longth of been Compa Cass Definite integrals equation 2. Days Day Beer Bas Pers Pos λ... Non-dimensional frequency parameter (ph 1/2) 1/2 Mon-dimensional normal and shear May X. buckling payanotors

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ABSTRACT

VIBRATION AND BUCKLING OF GENERALLY ONTHOTROPIC PLATES

by

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Indian Institute of Technology, Kanpur, SEPTEMBER, 1970.

Vibration and buckling characteristics of thin. rectangular plates, with arbitrary exientation of orthotropicity are studied. Deflections are assumed to be small and the effects of shear deformation and rotatory inertie are neglected. Approximate solution of the governing differential equation obtained by the principle of minimum potential energy using a 16 mode Maleigh Mits procedure. Beam character istic functions have been used as "admissible functions" to represent the plate deflections. The required integrals of these functions are evaluated and presented. (The variational equations so obtained are general in nature and are used to find the non-dimensional frequency and critical buckling load parameters of plates with different combinations of simply supported, clamped and free edges, by solving the correspondin eigenvalue problem. Amorical results of natural frequency and buckling loads are presented for Maple plymod plates having various arbitrary boundary conditions at the edges. different side ratios and angles of orthotropicity. The

results for the specially orthotropic case (e : 0° or 90°) are compared with previously published results and are found to be in close agreement. The stability and vibration characteristics of a simply supported plate with uniform normal implane lead on a pair of edges is studied. The buckling of simply supported plate subjected to shear leads at the edges is investigated and numerical results are presented for a Mahagony plywood plate.

Graphs are included to show the effect the angle of orthotropicity of the plate on the frequency and critical buckling leads.

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INTRODUCTION

1.1 CRMERAL

Orthotropic materials play an important role in modern technology. The conflicting demands of increase in strength and stiffness on the one hand and reduction in weight on the other have led to the use of laminated, stiffened or reinforced construction. Such structural elements are extensively being used in aircraft, missile and ship construction. In the past, materials, regardless of their composition and construction, were generally assumed to be homogeneous and isotropic because of the resulting simplification in the analysis. The present day sophisticated technology, however, requires that the static and dynamic behaviour of orthotropic structures be analysed fully. While certain materials like wood, are orthotropic by nature, a great variety of built up plateline structures exhibit artificial orthotropicity. Such materials have different elastic properties in different directions. They have, however, three mutually perpendicular planes of elastic symmetry. In an orthotropic plate one of the planes of symmetry is parallel to the plane of the plate. A rectangular plate is "specially orthotropic" if its sides are parallel to the remaining two planes of elastic symmetry, otherwise it is termed as "generally orthotropic". While the isotropic and specially orthotropic plates have received considerable attention of several authors, comparatively less work has been done in the case of generally orthotropic plate.

1.2 SURVEY OF LITERATURE

a. Vibration Problem:

The transverse vibration characteristics of thin rectangular isotropic plates were analysed and reported in the past. Timoshenko (1) developed the expression for the potential energy of bending and derived the governing differential equation based on the small deflection thin plate theory. Exact solutions of the differential equation exist when -

- 1) all edges are simply supported and
- ii) a pair of opposite edges simply supported with arbitrary boundary conditions at the other edges (method of levy).

varburton (2) determined approximate frequencies of rectangular isotropic plates, subjected to different combinations of free, supported or clamped edges. He applied the Rayleigh method, representing the deflections by suitable characteristic functions, which satisfy the boundary conditions. This method, however, yields the frequencies which are higher than the exact values. Young (3) used the Raleigh Ritz's method to find the approximate natural frequencies of isotropic rectangular plates by using the beam characteristic functions as "admissible" functions to represent the deflection. Samerical results were given

^{*}Numbers in the parentheses designate references at the end of the Thesis.

^{**}In this method the frequency is obtained by equating the maximum kinetic energy of the system to its maximum potential energy.

for square plates with

- i) all edges clamped,
- 11) one edge fixed and remaining free (cantilever)
 and 111) two adjacent edges clamped and the remaining free.

Barton (22) extended the treatment of Young to rectangular and skew cantilever plates with various side ratios. He also verified the results experimentally. The experimental values of frequencies were found to be less than those determined from theoretical analysis.

Lekhnitski (34) derived the basic equations of anisotropic elasticity and has considered at length many problems of stress distribution and deformation. Hearmon (5) considered the Booke's law in its most general form and gave the expressions for the stiffnesses and compliances of orthotropic plates in any arbitrary direction by coordinate transformation. Hearmon and Adams (6) compared the deflection pattern of this rectangular plates of metal and plywood (cut at various angles to the grain direction) as found from experiments with that from the theoretical analysis, when they are subjected to uniform bending and/or twisting moments at the edges. The results support the theory of bending and twisting of outhotropic plates. Hoppman and Baltimore (7) proposed that an isotropic plate stiffened by ribs can be treated as a homogeneous orthotropic plate by finding the equivalent elastic constants. Enough these elastic moduli it is possible to predict the behaviour of the plate subjected to any boundary conditions. The experimental results were in close agreement with the

theoretically predicted values.

Buffington and Hoppmann (4) used the Levy's method to find the natural fraquency of rectangular, thin, specially orthotropic plates with a pair of opposite edges simply supported and with arbitrary boundary conditions on the remaining edges. Frequency equations and modal eigen functions were derived. The arbitrary boundary conditions included various combinations of free, supported, clamped or clastically restrained edges. Hearmon (8) used the Baleigh's method to find the approximate frequencies of vibration of specially orthotropic plates under different combinations of clamped or supported edges. Closed form expressions were derived for the frequencies using beam characteristic functions to represent the deflection. Numerical results of the fundamental frequency of square plates were given for six combinations of supported and clamped edges. Someyajulu and Szinivasan (9) extended the method of Huffington and Hoppmann to find the first six frequencies of vibration of specially orthotropic plates with different side ratios. They, further, used the Raleigh Rits method to determine the first five frequencies of vibration of specially orthotropic captilever plates. Americal results of those frequencies were given for five materials with different orthotropic proporties and side ratios.

Calligeres and Dugundji (10) investigated the supersomic flutter of generally arthotropic panels using the principle of minimum potential energy and the Raleigh-Ritz method. They have plotted the frequencies of natural vibration for the first sixteen modies of such panels with three different side ratios and two sets of orthotropic properties.

weeks and Shidler (11) considered the vibration characteristics of thin, rectangular, specially orthotropic plates, with implane loads, subjected to different combinations of supported, clamped and elastically restrained edge conditions. The Gelerkin's method was used to derive the frequency equations. Dickinson (12) used a sine series solution to analyse the free vibration of specially orthotropic, thin rectangular plates. The method of solution is applicable to plates with any edge conditions expressible using sine series. Application to plates with the following boundary conditions was given:

- A pair of opposite edges simply supported and each of the remaining edges being simply supported, free, clamped or partially restrained.
- 11) all edges clamped and
- iii) two opposite edges clamped and the remaining edges free.

Numerical results were given for square plates with (i) all edges clamped and (ii) two opposite edges clamped and the remaining being free. The results were compared with those already available in the literature. The accuracy of the numerical results depends on the convergence of the roots of the determinental equation.

b. Stability Problem:

The buckling of isotropic plates was discussed at length by Timosheuko and Gere (13). The governing differential

equation expressions for potential energy were derived. Exact solutions for the critical buckling loads were given for plates with all edges simply supported. The governing differential equation was also solved for plates with the pair of loaded edges simply supported and with arbitrary boundary conditions at the remaining edges, using Levy's method. Maulbetsch (14) used the energy method to find the approximate critical buckling loads of rectangular isotropic plates with clamped edges. Levy (15) obtained an exact solution of the governing differential equation interes of infinite series to get the critical buckling loads of isotropic plates with clamped edges. The accuracy of the results depends on the convergence of the series and he estimated that the error is of the order of 0.1%. Green and Hearmon (16) derived the differential equation of bending of thin, rectangular, generally orthotropic plates with uniform inplane loads. Using Pourier series method, the cases of a plate with (1) all edges simply supported and (11) all edges elamped were solved. Results were also given for a plate with a pair of opposite edges simply supported, the remaining being clapped. Imported values of critical anial and shear buckling loads were evaluated for square and infinitely long generally orthotropic plates with support conditions (1) and (11). They remark that the results obtained are reasonably accurate for square plates but as the side ratio (a/b) increases the accuracy diminishes.

^{*}Minimizing the potential energy.

Das (17) used the Levy's method to find the critical buckling loads of thin rectangular specially exthetropic plates with the pair of loaded edges simply supported and the remaining edges having arbitrary boundary conditions. Numerical results were given for different types of plywood plates under the following boundary conditions:

- 1) all edges simply supported
- ii) three edges simply supported and the fourth clamped and
- iii) three edges simply supported and the fourth free.

Lure (18) has observed that the vibration as well as the buckling analysis of thin rectangular plates leads to the same Eigenvalue problem under certain boundary conditions. He has suggested a method of finding the frequency of natural vibrations from the critical buckling lead parameters. Someyajulu and Srinivasan (9) used this analogy to find the critical buckling leads from the frequency data. Weeks and Shidelr (11) calculated the buckling characteristics of specially orthotropic plates by noting the fact that the critical buckling lead is the lowest implane lead which makes the frequency of transverse vibration of the plate (subjected to implane lead) Vanish.

1.3 STATEMENT OF THE PROPLEM

In the present investigation the vibration and buckling characteristics of thin, rectangular gener/ally exthetropic plates are studied. The material is assumed to be linearly electic and the analysis is based on the small deflection theory

of thin plates. Effects of shear deformation and retatory inertia are neglected. A sixteen mode Raleigh Ritz procedure along with the principle of minimum potential energy is used to get an approximate solution of the governing differential equation. Beam characteristic functions, which represent the normal modes of vibration of slender beams, are used as "admissible" functions.

Integrals of these functions are evaluated using a numerical integration scheme. A general formulation of the problem was obtained which could be used to find the frequencies of natural vibration as well as the critical buckling leads (shear and normal) of thin rectangular plates with arbitrary orientation of orthetropicity and any boundary conditions at the edges.

The above procedure is applied to find the nondimensional frequency and critical buckling load parameters of rectangular generally orthotropic plates having the following edge conditions:

- 1) all edges simply supported,
- 11) all edges clamped,
- iii) one edge clamped and the rest free (cantilever plate),
 - iv) three edges simply supported and the remaining free and
 - v) a pair of opposite edges simply supported and the rest clamped.

PORMULATION AND SOLUTION

2.1 GENERAL EQUATIONS FOR ORTHOTROPIC PLATES

conditions of a generally orthotropic, thin, rectangular plate are derived by applying the principle of minimum potential energy. The plate (figure No.1) is assumed to have three axes of elastic symmetry, one at right angles to the plane of the plate and the other two lying in its plane, making an angle 6 with the sides. The small deflection theory of thin plates is utilised in the analysis which is based on the following assumptions.

- 1. Thickness of the plate is small when compared to its lateral dimensions.
- 2. Deflection of the middle surface is small when compared to the thickness of the plate.
- 3. Rectilinear sections which in the undeformed plate were normal to the middle surface remain rectilinear and normal to the bent middle surface and
 - 4. Normal stress T_g on planes parallel to the middle surface is small in comparison with the stresses T_{10} T_{2} and T_{n} acting in its plane.

Under these assumptions the displacements are linearly related to the distance x from the middle surface of the plate

and are given by

$$u = -\frac{3}{2}\frac{\pi}{4}; \qquad v = -\frac{3}{2}\frac{\pi}{4}$$

the strains in the plane of the plate are given by

$$26 = \frac{9}{3} + \frac{3}{3} = -2 = \frac{3}$$

The stress displacement relations are (from eqn. A.5)

The stress components T4 and T5 are given by (ref. 8)

The bending moments, twisting moments and implace loads per unit length are given by

 $N_1 = \int_{0}^{1/2} T_5 ds and N_2 = \int_{0}^{1/2} T_4 ds \dots 2.4$

where h is the thickness of the plate.

Substituting equations 2.2 and integrating we get

$$M_{12} = (D_{11} \frac{\partial^{2}_{12}}{\partial x^{2}} + D_{12} \frac{\partial^{2}_{12}}{\partial y^{2}} + 2 D_{16} \frac{\partial^{2}_{12}}{\partial x^{2}})$$

$$M_{2} = (D_{12} \frac{\partial^{2}_{12}}{\partial x^{2}} + D_{22} \frac{\partial^{2}_{12}}{\partial y^{2}} + 2 D_{26} \frac{\partial^{2}_{12}}{\partial x^{2}}) \dots 2.5$$

$$M_{12} = (D_{16} \frac{\partial^{2}_{12}}{\partial x^{2}} + D_{26} \frac{\partial^{2}_{12}}{\partial y^{2}} + 2 D_{66} \frac{\partial^{2}_{12}}{\partial x^{2}})$$

and the plate shears are given by (from equation 2.3)

$$Q_2 = -\left[D_{12} \frac{\partial^2 y}{\partial x^2} + 0 D_{16} \frac{\partial^2 y}{\partial x^2} + (D_{12} + 2 D_{66}) \frac{\partial^2 y}{\partial x^2} + D_{26} \frac{\partial^2 y}{\partial x^2} + D_{26} \frac{\partial^2 y}{\partial x^2}\right]$$

The above equations for the specially exhibitropic case are obtained by putting $b_{16} = b_{26} = 0$ and for the isotropic case by putting $b_{11} = b_{22} = 3/(3-7^2)$ and $b_{66} = 6$.

Substituting equations $A_i > 1$ in $B_i > 1$ the rigidities of a generally orthotropic plate $\{D_{i,j}\}$ with an angle of orthotropicity $A_i > 1$ on the expressed in terms of the rigidities of a specially orthotropic plate by a set of equations obtained by replacing $A_{i,j} > 1$

by Dij and bij by Dij in equations 8.7 where Dij are the rigidities of a specially orthotropic plate.

The total potential energy (U) of a thin, rectangular, plate of uniform thickness h undergoing transverse vibrations is given by (ref. 19).

$$U = \frac{1}{2} \int_{-h/2}^{h/2} \int_{0}^{a} \int_{0}^{b} (T_{1}s_{1} + T_{2}s_{2} + T_{3}s_{3}) dxdyds$$

$$-\frac{1}{2} \int_{0}^{a} \int_{0}^{b} \left[yw - H_{1} \left(\frac{\partial w}{\partial x} \right) - H_{2} \left(\frac{\partial w}{\partial y} \right) - 2H_{12} \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \right] dxdy$$

$$\dots 2.6$$

Substituting the stress-strain (\wedge .5) and the strain-displacement (2.1) relations we get

$$\int_{0}^{\infty} \left[D_{11} \left(\frac{\partial}{\partial x} \right) + D_{12} \frac{\partial}{\partial x} + D_{22} \left(\frac{\partial}{\partial x} \right) \right] dx dx$$

$$= \left(\frac{\partial}{\partial x} \right) + \left(\frac{\partial}$$

The differential equation and the boundary conditions of the plate can be obtained by applying the "Principle of minimum potential Energy". Taking the variation of equation 2.8 and performing the integration by parts, we get

$$+ \sum_{n} \left[\sqrt{3} + \frac{9^{\frac{n}{2}}}{9^{\frac{n}{2}} + 1} + \sqrt{3} + \sqrt{3$$

where M_1 , M_2 , M_{12} , Q_1 and Q_2 have the same meaning as in equations 2.5 and 2.6. Since $\delta w(x,y,t)$ is arbitrary, equation 2.0 will be satisfied if the following conditions hold.

... 2.11

44 2.10

This is the governing differential equation of a vibrating plate subjected to implane loads N1, N2 and N12 as shown in figure 1.

11)

a) Along edges x = 0, a

(1) Either
$$M_1 = 0$$
 or $\delta(\frac{\partial w}{\partial x}) = 0$

b) Along edges y . o, b

c) At corners

This condition can be phipically interpreted to give the boundary conditions of the plate at the edges.

For a specially orthotropic plate the terms containing D_{16} and D_{26} vanish and the Coverning differential equation (2.10) is simplified accordingly.

For the isotropic plate with no inplane leads the equation reduces to the well known form

where D . Eh³/12(1+)²)

For a plate undergoing transverse vibrations without any external loads the governing differential equation is obtained by replacing

p by $-3\frac{\partial^2 v}{\partial t}$ and substituting $N_{12}N_{2} = N_{12} = 0$ in equation 2.11. It then becomes

D₁₁
$$\frac{\partial^2 V}{\partial x^2} + 2D_{12} \frac{\partial^2 V}{\partial x^2} + D_{22} \frac{\partial^2 V}{\partial x^2} + A^2 D_{16} \frac{\partial^2 V}{\partial x^2} + D_{26} \frac{\partial^2 V}{\partial x^2} + B^2 \frac{\partial^2 V}{\partial x^2} = 0$$
 ... 2.13

This equation is difficult to be shived in most cases and exact solutions are known only for some simple cases. For the case of an isotropic or specially exthetropic plate with all edges simply supported, the equation has been solved (ref. 1 and 19). Exact solution also exists when a pair of epposite edges are simply supported for these plates (Levy's method). In this method the deflection is assumed to be

equation 2.14 when substituted in the partial differential equation, equation will convert it into an ordinary differential equation. The frequency equation can then be derived by solving this equation. Buffington and Reppendent (4) have used this method for specially orthotropic plates.

Por other boundary conditions of isotropic and specially orthotropic plates and any boundary conditions of generally

equation exactly. Under these circumstances we have to take recourse to approximate methods. Raleigh's method was used by Warburton (2) for isotropic plates and by Hoppmann and Baltimore (7) for specially orthotropic plates. This method, however, cannot be applied to the generally orthotropic plates because of the occurence of terms containing D₁₆ and D₂₆. For the same reason Gelerkin's method also cannot be applied for this case. In the present study the Raleigh Ritz method is used to obtain approximate solution to the governing differential equation. This method was used by several previous authors. Young (3) applied it to isotropic plates, Somayajulu and Srinivasan (9) for specially orthotropic plates and Calligerows and Dugundji (10) to the analysis of orthotropic panel flutter.

2.3 THE RAISIGH RITZ METHOD.

tional problem for the usual characteristic value problem. In a variational problem it is not necessary to impose boundary conditions in advance, in order to single out a specific solution.

The vanishing of the first variation of the functional not only implies the Buler's equations, but also the natural boundary conditions. Courant (23) remarks that for rigid boundary conditions the approximation of the Raleigh Ritz method is good and a few admissible coordinate functions would in most cases, suffice to yield the desired convergence. But the boundary conditions impose a restriction on the choice of functions to represent the deflection. For free boundaries, however, the choice of functions is unlimited, but the convergence is rather slow and it

becomes necessary to take more terms in the series to get the desired accuracy.

In applying the Raleigh Ritz method, the deflection of the plate is assumed as a linear series of admissible coordinate functions. The coefficient of each term of the series is adjusted so as to minimize the potential energy (V) of the plate. The deflection V of the plate can be assumed as

$$W (x,y,t) = \sum_{m=1}^{p} \sum_{m=1}^{q} C_{mn} x_{m}(x) x_{n}(y) e^{2Gt} \dots 2.15$$

where the functions X_m and Y_n are "admissible" functions i.e. they satisfy the artificial (or rigid) boundary conditions and need not satisfy the natural boundary conditions. In the case of plates prescribed values of slope and deflection constitute the artificial boundary conditions and the values of moments and shear force constitute the natural boundary conditions. Better conversionable force, however, can be obtained if the natural boundary conditions are also satisfied by the assumed functions.

By substituting equation 2.15 in equation 2.9 the total Potential Energy U can be expressed as a function of the coefficients $C_{\rm min}$

for minimum potential energy we have $\delta u = 0$

Equations 2.16 represent a system of homogeneous. linear equations in the unknown quantities Com. There are p x q equations in p x q unknowns, which can be determined by equating the determinant of the coefficient matrix to sero. The Eigenvalues of the system can be determined by any numerical technique, using a high speed electronic digital computer.

2.4 PLATE SIMPLY SUPPORTED ON ALL EDGES.

For a plate with all edges simply supported the deflection can be assumed as

$$w(x, y, t) = \sum_{n=1}^{p} \sum_{n=1}^{q} c_{nn} \sin \frac{n\pi x}{n} \sin \frac{n\pi y}{n} \sin \frac{n\pi y}{n} e^{165t}$$

This satisfies the geometric boundary conditions $w(x_1,0,t) = w(x_1,0,t) = w(x_2,0,t) = w(x_3,0,t) = 0$

But, the natural boundary conditions

are not satisfied.

Substituting 2.17 in 2.9 and noting that for the

vibration problem $p = -\frac{\partial^2 v}{\partial t^2}$ we get, after simplification,

 $\frac{1}{2} \sum_{n} c_{nn} \cos \frac{n \pi x}{n} \cos \frac{n \pi x}{n} \cos \frac{n \pi x}{n} (\sum_{n} \sum_{n} c_{nn} \sin \frac{n \pi x}{n} \sin \frac{n \pi x}{n})$ $cos \frac{n \pi x}{n} \cos \frac{n \pi x}$

By substituting equation 2.18 in equation 2.16, carrying out the differentiation, simplifying and using the following integrals

and $\int_{\mathbb{R}} \sin \frac{n\pi x}{n} \cos \frac{n\pi x}{n} dx = 0$ if m + n is even $= \frac{20 \pi}{7(m^2 - n^2)}$ if m + n is edd

we get

$$(m + 1, 2 \dots p; n + 1, 2 \dots q)$$
 2.19

whore

$$D_{196} = D_{32} + 2 D_{66}, \qquad Z = 6^{\circ} 3 C^{\circ} + 2^{\circ} D_{33}^{\circ}$$
 $B_{3} = B_{1} C^{\circ} + 2^{\circ} D_{33}^{\circ}, \qquad B_{2} = B_{2} C^{\circ} + 2^{\circ} D_{33}^{\circ}$
 $B_{32} = B_{12} C^{\circ} + 2^{\circ} D_{33}^{\circ}, \qquad B_{33} = 2^{\circ} C C^{\circ} + 2^{\circ} C^{\circ}$

$$K_{\text{mars}} = \frac{16 \text{ mars}}{\pi^2 (r^2 + n^2) (s^2 - n^2)}$$
 when $n \pm r$ is odd

=0, when mir is even or nis is even

The torsional and flexural rigidities D_{1j} for any angle of orthotropicity, 6 can be obtained from equations 2.7 and A.9. For the free vibration problem $(R_1 * R_2 * R_{12} * O)$, it is convenient to write the set of equations in the matrix form

$$[K] \{e\} = Z[X] \{c\} = 0$$
 ... 2.20

K | will be adiagonal matrix when e = 0 or 90 and for other angles it will be real, symmetric and positive definite. The Sigenvalues of K which will be real and positive represent the natural frequencies of transverse with vibration of the plate. The Jacobi's method is used for solving the eigenvalue problem represented by 2.20 This is a method of diagonalization by successive rotations and iterates to all eigenvectors and eigenvalues simultaneously. This procedure consists of multiplications by matrices, which are similar in form to coordinate transferention matrices that represent angular retations. The successive multiplications result in the gradual increase of the diagonal terms at the expense of the off-diagonal elements. When finally the off-diagonal elements become soro, the diagonal terms of the resulting matrix are the eigenvalues and the continuous product of rotation is the model matrix. The method is readily applicable to real and symmetric matrics using the

high speed electronic digital computer.

2.5 PLATE WITH AMBITRARY BOUNDARY CONDITIONS.

For a plate with arbitrary boundary conditions at the edges we can assume

$$v(x,y,t) = \sum_{m=1}^{p} \sum_{m=1}^{q} c_{mn} X_{m}(x) Y_{m}(y) e^{i\omega t} ... 2.21$$

where $X_{\rm m}$ and $X_{\rm m}$ are both "admissible" functions i.e. they satisfy the rigid boundary conditions. In the present analysis the "beam characteristic functions" are used for these functions. These represent the normal modes of vibration of uniform, long and slender beams. They are were used as admissible functions for plate vibration problems by several previous authors Warburton (2) and Young (3) used them for isotropic plates and Hearmon (8) and Somayajulu and Srimivasan (9) for specially orthotropic plates. These functions, because of their orthogonal property, are simple to use and many of the Integrals required for the compertations for certain boundary conditions are calculated and tabulated by Young (3). The other integrals required for the generally orthotropic plate for all boundary conditions are evaluated and tabulated in the present investigation.

Beam Characteristic Functions:

These functions are obtained from the solution of the differential equation governing the transverse vibration of a uniform beam. The general form of these functions is given by $S_0 = \operatorname{Coch} \ B_0 \times + \mathbb{R} \operatorname{Coc} \ B_0 \times + \mathbb{R}$ (Sinh $B_0 \times + \mathbb{R} \operatorname{Sin} \ B_0 \times) \dots 2.22$

These are, thus, an infinite number of functions corresponding to c = 1,2 ... on representing the different modes of vibration. While the equation 2.22 represents the general form of these functions, for any particular boundary conditions of the beam the corresponding values of E,F, B, and L, are to be substituted. These functions and constants are given by Bishop (21), Hearmon (5) and Young (3). The data compiled from these references is given in table 1 for different boundary conditions.

These functions possess the important property of orthogonality i.e. for any two functions ϕ_m and ϕ_n

and

where I is the length of the bean

The characteristic functions for a simply supported been, which are not given in the table, can be taken as

These functions also satisfy the orthogonal relations 2.23 and 2.24, if we define $B_{\rm c} = c \, \pi \, (c = 1, 2 \dots c c)$.

The lowest mode (c * 1) in table corresponds to two model lines and when dealing with free-supported and free-free plates modes of lower frequencies corresponding to rigid body translation and rotation are possible. These modes are represented by additional characteristic functions proposed by Warburton (2) -

1) Free-supported plate

11) Free-free Plate

$$\theta_2 = \sqrt{3} (3 + 2x/\ell)$$
 ... 2.27

These functions also satisfy the orthogonal relations represented by equations 2.23 and 2.24 if we define for these lower modes $B_0 = 0$ (c = 1,2)

Using these functions for $X_m^{(x)}$ and $Y_m^{(y)}$ in equations 2.21 the total potential energy of the vibrating plate with any combination of free, supported or clamped edges can be expressed in terms of the unknown constants C_{min} . Substituting 2.21in equations 2.8 and 2.15, carrying out the differentiation and simplifying using the following notation

where

δ₀₀ : 1 4° n = 0

. . 40 24 2

and χ^2 . ph w a4 / Des

For the free vibration problem (N₁ = N₂ = N₃₂ = 0) the equations 2.30 can be written in the matrix form

$$[K] \{c\} * \lambda^2 [I] \{c\} * \cdots 2.31$$

The matrix [K] is real and symmetric irrespective of the boundary conditions at the edges of the plate. The eigenvalue problem represented by the above equation can be solved to yield the frequencies of transverse vibration of the plate (λ) for any boundary conditions at the edges, angle of orthotropicity (e) and side ratio (R).

2.6 SUCKLING OF GENERALLY ONTHOTROPIC PLATES

The critical values of ferces applied in the middle plane of the plate at which the flat form of equilibrium becomes unstable and the plate begins to buckle can be determined by several methods.

- 1) The plate is assumed to have initially some curvature or lateral load. The implame forces required to make deflections tend to grow indefinitely are the critical values (Imperfection method).
- 2) The plate is assumed to buckle slightly under the action of the middle plane forces and the values of these forces are calculated in order to keep the slightly buckled shape in equilibrium method). The differential equation of the surface is obtained in this case by putting p = 0 in equation and

$$D_{11} \stackrel{\partial^4_{12}}{\partial x^2} + 2D_{126} \stackrel{\partial^4_{12}}{\partial x^2} = D_{22} \stackrel{\partial^4_{12}}{\partial y^2} + 4D_{16} \stackrel{\partial^4_{12}}{\partial x^2} = D_{22} \stackrel{\partial^4_{12}}{\partial y^2} + 4D_{16} \stackrel{\partial^4_{12}}{\partial x^2} = D_{22} \stackrel{\partial^4_{12}}{\partial x^2} = D_{22} \stackrel{\partial^4_{12}}{\partial x^2} = D_{23} \stackrel{\partial^4_{12}}{\partial x^2$$

If only N₁ is acting, the minimum value of N₁ which satisfies the above equation and the boundary conditions of the plate, is the critical buckling load. This method has been used in the literature for the buckling analysis of plates. Green and Hearmon (16) have used this method for 'generally' orthotropic plates employing the Fourier series method to solve the differential equation Das (17) has used this approach to find the critical buckling loads of rectangular, specially orthotropic plates, using the Levy's method to solve equation 2.32.

3) Where a rigorous colution of equation 2,32 is unknown we can calculate the approximate critical buckling loads by equating the strain energy of bending to the work done by the implane forces (Emergy method).

In this mothod the plate is assumed to undergo small lateral bending consistent with the boundary conditions. If the work done by the implane forces is smaller than the strain energy most bending for every possible shape of lateral buckling, the flat form of equilibrium is stable. If it is greater than the strain energy of bending the plate is unstable in the flat form

and undergoes buckling. The implane loads reach critical values when the work done by the implane forces is equal to the strain energy of bending. Timoshenko and Gore (13) have used this method to find the critical buckling loads for rectangular, isotropic plates with all edges simply supported.

- 4) The lowest implane load which makes the frequency of transwise vibration vanish gives the critical buckling load (Kinetic method). This can be obtained from a plot of the frequency of transwise vibration with implane load versus the magnitude of the implane load. This method was used by weeks and Shideler (11) to find the critical buckling loads of specially orthetropic plates.
- 5) By noting the fact that under certain boundary conditions the buckling and vibration problems are similar Boundary value problems, the results of one can be obtained if the results of the other are available. The critical buckling load parameters can be found from the natural frequency parameters by suitable commission. This analogy can be clearly seen in the case of orthotropic plate with simply supported edges from equation 2.17. For the vibration problem, in the above equations we have $R_1 + R_2 + R_{12} + C$, whereas for the buckling problem we have $R_2 + R_{12} + C$. The equation can then be put in the matrix form

Thus both the problems can be reduced to the same Eigen value problem and the Eigenvalues, 2 can be interpreted as the

natural frequencies for the vibration problem and as the critical buckling loads for the buckling problem.

Such a similarity can also be noticed in the case of a plate with the pair of loaded edges simply supported. Again we have for the vibration problem $R_1 = R_2 = R_{12} = 0$ and for the buckling problem $R_2 = R_{12} = 0$. It can be seen that $D_{\rm mr} = (m\ w)^2\ \delta_{\rm mr}$ and the eigenvalue problem can be put in the matrix notation (equation 2.33). The Rigenvalues can be suitable interpreted as

2 * (Prada 4/D1 22) for the vibration problem

. No a2 m2 w2/ D1 for the backling problem.

2.7 CRITICAL BUCKLING LOADS.

Lord material parts of the transfer of the state.

In the present study the critical buckling load parameters of himiz thin rectangular generally orthotropic plates are found by the equilibrium method. An approximate solution to the differential equation 2.32 is obtained by the principle of minimum potential energy using the Raleigh Rits procedure. Characteristic beam functions are used as admissible functions to represent the deflection of the plate. The analogy shown in the previous section between the vibration and buckling problems can be used to find the buckling loads from the frequency data when the plate has all edges simply supported. For arbitrary boundary conditions, however, the eigenvalue problem is to be solved separately.

1) UNI-AXIAL BUCKLING

For a plate with arbitrary boundary conditions at the edges subjected to inplane compressive lead parallel to x-axis only, we can put $\lambda = N_2 = N_{12} = 0$ in equations 2.30. They can then be put in the matrix form

$$[K] \{C\} - X_{0}[G] \{C\} = 0$$
 ... 3.34

where X₀ = N₁ e²/ D¹22

Premultiplying by $[G]^{-1}$ we get $[A]\{C\}$ to where [A] = $[G]^{-1}[K]$

A is a real non-symmetric matrix and its eigenvalues represent the critical buckling leads for different modes of deformation of the plate. Generally, the lowest critical buckling lead is of interest and is determined by finding the largest eigenvalue of $[A]^{*2}$ = $[R]^{*2}$ [G]

11) BI-AAXIAL BECKLING.

The minimum compressive lead papallel to meanis to cause buckling of a plate subjected to another implane lead parallel to yearis (on the other two edges) can be determined by a procedure similar to the one used in (i). Suitable numerical values are substituted for B₀ in equations 2.30 and the eigenvalue problem is solved exactly in the same manner.

111) SEEAR BUCKLING.

The minimum critical sheer loads along the edges of the

plate to cause buckling of their rectangular generally orthotropic plates can be determined by putting λ , N_1 , N_2 =0 in equations 2.30 which can then be put in the form

$$[K]\{C\} = X_S[O']\{C\}$$
 ... 3.36

where X = N 12 a2/ D22

The matrices [K] and [G'] are real and symmetric and the eigenvalue problem can be solved exactly in the same manner as in (i) and (ii).

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MUMBRICAL RESULTS AND DISCUSSION

3.1 FREQUENCY OF TRANSVERSE VIBRATION.

1) PLATE SIMPLY SUPPORTED AT ALL EDGES: The natural frequencies of transverse vibration of a thin rectangular generally orthotropic plate are calculated by zk solving the Eigenvalue problem represented by equation 2.20. Americal results are obtained for a maple plyfood plate for which the elastic properties are B_{11}/B_{126} = 1.543 and B_{22}/B_{126} 4.813. The rigidities Dal, Dee, Die and Dee are calculated from equations A.9 and 2.7. The elements of the matrix K are evaluated from equations 2.19 taking p = q = 4 for different side ratios (a/b) and angles of orthotropicity (6). For the specially orthotropic case (i.e. 6 = 0 or 900) H is a 16 x 16 diagonal matrix and for the generally orthotropic plate (for other values of e) the orthotropic coupling terms containing Dag and Dog contribute the off-diagonal terms. However, the diagonal terms are large when compared to others and the satrix is real and symmetric. The eigenvalues of K are evaluated by the Jacobi method. The non-dimensional fragmency parameter \ . (Ph w a2 b2/ m Dist 1/2 is enloulated for the first sixteen modes of vibration. These modes are characterized by m and n (in equations 2.19) which represent the number of half waves into which the plate bends along the x and y directions respectively. The frequencies of the first six modes of vibration are given in table 4 for various side ratios and angles of orthotropicity.

The results for the particular cases when 0 = 00 and 900 are compared with those obtained from the closed form empression derived by Hearmon (5) using the Releigh's method in Table 3. This method gives the exact frequency for the simply supported case. The comparison shows that the results of the present investigation agree exactly (upto three decimal places) with those of Hearmon. The Releights method, however, cannot be applied for other angles of orthotropicity because of the terms Dig and Dog in equation 2.13. The effect of the side ratio on the frequency of the two modes (m = 1, n = 1 and m = 1, n = 2) is shown in figure 2. The frequency decreases rapidly with the increase of the side ratio from a high value, attains a minimum at a particular a/b ratio and increases at a less rapid rate. The value of the side ratio at which the frequency is minimum depends upon the mode shape the angle of orthotropicity and the clastic properties of the plate. Figure 3 shows the variation of the frequency of the first three modes (m . 1, n . 1, m = 1, n =2 and n = 2. n = 1) with the angle of orthotropicity for side ratios of 0.4, 1.0, 2.0 and 3.0. For cortain modes the frequency increases with the angle of esthetropicily while for others it decreases. There are some modes for which the maximum frequency occurs at a value near about 45°.

The frequency of transverse vibration of a plate subjeted to normal implane head along a pair of opposite edges is calculated from equation 2.20. Americal values (both tensile and compressive) are substituted for N₁ in equations 2.30 heaping N₂ = N₃₂ = 0 in them to evaluate the elements of the matrix

K, which is real and symmetric. The eigenvalue problem is solved and the frequency parameters are determined and given table 5 for various side ratios, angles of orthotropicity and edge loads. Figure 4 shows the effect of the implane load on the frequency of transverse vibration. The square of frequency decreases linearly with the decrease of the tensile load (or increase of compressive load) and the implane load corresponding to zero frequency is the static buckling loads of the plate.

11) PLATE WITH ARBITRARY BOUNDARY CONDITIONS.

The frequencies of transverse vibration of thin rectangular plate with arbitrary boundary conditions at the edges are found by solving the eigenvalue problem represented by equation 2.31. The numerical values of the integrals required in equation are determined by the integration of the beam characteristic functions. These characteristic functions which represent the mode shapes of lang, slender beams subjected to transverse vibrations for different boundary conditions are given in table 1.C. represents the mode number (the number of half waves into which the beam bends). A numerical integration procedure using the "Romborg Integration" is utilized to evaluate all the required integrals. The mumerical results are presented in table 2 (a, b and c). The integrals $\int p^2_{m} p_{n} dx$ and $\int g_m^{11} g^1$ dx were not previously reported in the literature whereas the integrals $\int g^2_{pp} g^3_{p}$ dx and $\int g^{22}_{pp} g_{p}$ dx were tabulated by Young (3) for clamped- clamped, clamped - free and

cases are in close agreement with those of reference (3). This numerical data is now substituted in equations 2.30, to evaluate the elements of the matrix K. This matrix is real and symmetric irrespective of the boundary conditions of the plate. The eigenvalues are determined as in case (i) and the dimensional frequency parameters (χ^2 Ph χ^2 a⁴/D¹22) are evaluated for the first 16 modes of vibration. A general computer program is made (given in Appendix B) to find the frequencies of vibration of thin rectangular plates with any arbitrary boundary conditions at the edges, with side ratios ranging from 0.5 to 3.5 (in steps of 0.5) and with the angle of esthetropicity ranging from 0.0 to 90° (in steps of 15°).

(Continued...)

Numerical Values of these frequencies for a maple plywood plate (B_{11} / B_{22} = 3.12, B_{12} / B_{22} = 0.1206 and B_{36}/B_{22} = .2637) are evaluated and presented in tables 6a - 6e for the following boundary conditions;

- a) all edges simply supported
- b) all edges clamped
- two opposite edges clamped and the others simply supported
- d) one edge clamped and the rest free (canti lever plate)
- and e) one edge free and the rest simply supported.

while the frequencies of generally enthotropic plates with arbitrary boundary conditions are calculated for the first time, the specially enthotropic case was solved by Hearmon (8) using the Raleigh's method for their rectangular plates having various combinations of clamped and supported edges. The fundamental, frequencies found from equation 8.31 are compared with those determined from the closed form expressions derived by Hearmon for specially enthotropic case i.e. when 8 * 0° and 8 * 90°.

The Aminigh's method is known to give exact frequency for the case of a plate simply supported at all edges, whereas, for other boundary conditions it gives an upper bound for the frequencies. Comparison in table 7 shows a very good convergence of the Baleigh Mitz method to these true values for the sess plate from the higher side (the difference being of the order of 0.08%). For plates with other boundary conditions, for which comparison

could be made, it can be seen that the frequencies found in the

present investigation are less than (of the order of 0.2%) those given by Raleigh's method. This indicates that the Ritz's modification of the Raleigh's method improves the convergence to the true value from the higher side considerably. In table 8 the first five frequencies of cantilever plate obtained from equation are compared with these given by Somayafulu and Srinivasan (9) for the case when 8 . O. The values in reference 9 appear to have been wrongly tabulated the fundamental frequency for side ratios 0.5 and 2.0 having been emitted. Comparison is made after the necessary correction and values agree very closely. This is to be expected since the equations 2.30 become exactly same as those used in reference 9 for angles of orthotropicity of 0 or 900. The a/b ratio has practically no effect on the fundamental frequency of vibration of a specially orthotropic cantilever plate. Barton (22) reports a similar behaviour of a rectangular isotropic plate.

The fundamental frequency of vibration is plotted against the angle of orthotropicity in figure 5 and 6 for side ratios (a/b) 0.5, 1.0, 2.0 and 3.0 and various boundary conditions. The frequency of clamped - clamped plate reduces with the increase of the angle of orthotropicity, 6 for a side ratio of 0.5 whereas for other ratios it decreases. The occupiate has the highest frequency and the cantilover plate the lowest. The frequencies of ecce and seec are wide apart for low a/b ratios and they come closer as this ratio is increased.

3.2 CRITICAL BUCKLING LOADS.

1) PLATS SIMPLY SUPPORTED ON ALL EDGES:

The critical buckling loads of generally orthotropic plates, when all edges are simply supported, can be determined from equation 2.33 by solving the corresponding eigenvalue problem.

a) Uniaxial buckling: For buckling under uniform compressive implane loads on a pair of opposite edges the eigenvalue problem is exactly same in 3.1 (i). The elements of the matrix k are found from equations 2.19 by putting " = Ro = Rip = 0. The matrix K is found to be real and symmetric and the eigenvalues are determined using the Jacobi's method. The non-dimensional critical buckling load parameters (Ke . b2 N/ h3 NL) are evaluated for mahogany plyfood plate for which Bal Boo = 3.04. D_{126} P_{22} = 0.438 and E_{L} = 1.35 x 10^6 psi. These values for the first three modes of deformation of the plate for various angles of orthotropicity and side ratios are given in table 9. Green and Hearmon (16) have, by using a Fourier Series solution, found the critical buckling loads of generally opthotopic plates and have presented numerical results for a square mahageny plywood plate. They have made approximations by limiting the number of terms in the series to six and simplifying the results to derive closed form expressions for the buckling loads. It is remarked that the results are resembly accurate for a side ratio of 1 and the accuracy decreases with the increase of the side ratio and the magnitude of the orthotropic coupling terms

(terms containing Dis or Dos). The present investigation takes 16 term series for the deflection function and is expected to give better results. These results are compared with those of Green and Hearmon in table 9 for a square plate and are found to be in close agreement. In figure 7 the minimum critical buckling load is blotted against the a/b ratio for various angles of orthotropicity. The behaviour is similar to that of specially orthotropic plates as reported by Das (17). For low values of side ratios the first mode gives the minimum critical buckling load and as a/b ratio increases beyond certain value (indicated by a Kink in the curve) the second mode give the minimum buckling load. The angle of orthotropicity alters the minimum buckling loads and the location of the kinks along the curve. For certain a/b ratios the advantage in having a value of 8 other than 0° or 900 for increasing the critical buckling load is clearly soon in figure ?.

b) Bi-axial Buckling: The minimum estitical normal load parallel to y-axis on two opposite edges required to cause buckling of a thin rectangular generally estimate pie plate, when it is subjected to uniform normal implant load parallel to the x-axis on the remaining two edges, is determined from equation 2.20. The elements of the matrix E are determined from equations 2.19 by giving suitable values to B₂ keeping 2 = B₁₂ = 0. The matrix E is real and symmetric and the elements are determined by the Jacobi's method. The numerical

results for mahogony plywood plate (D11 / B22 a 3.04 and $B_{126}/B_{22} = 0.438$) are given in table 9. The critical buckling loads for the first three modes of deformation are plotted against the implane load parallel to the y-axis in figure 4. The buckling load (along y-direction) is found to very linearly with the uniform implane load acting parallel to the y-axis. A plate loaded in tension requires a higher compressive load to eause buckling of the plate than that required by an unloaded plate. A compressive load, on the other hand, requires a smaller load to buckle the plate. Timeshenko (13) describes a similar behaviour of isotropic plates under uniform edge loads in two perpendicular. The lines in figure 4 corresponding to the modes m = 1, n = 2 and m = 2, n = 1 are not symmetrical with reference to the R1 and R2 axes whereas for the isometric case such a symmetry exists for these and other similar modes.

c) Buckling Under Shear Loads: The minimum shear loads along the edges of a thin rectangular plate required to cause buckling can be determined from equations 2.19 by pulling 2. R₁ . R₂ . O. The lowest value of R₁₂ which makes the determinant of the coefficient matrix of equations 2.19 equal to zero is the minimum critical buckling load. Starting from a value of zero for R₁₂, it is increased gradually in small steps and the sign of the determinant is found, when the sign changes the step size is reduced and the process is repeated until a value of R₁₂ is found to sufficient accuracy, which makes the determinant zero very nearly. This value is the shear buckling

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load. The numerical results obtained for the mahogony plywood plate are shown in table 11. The results are compared in table 10 with those of Green and Hearmon (16) for the case when a/b = 1.0 and are found to be in close agreement. Figure 9 shows the variation of the shear buckling loads (*ve and -ve) with the angle of orthotropicity.

11) PLATE WITH ARBITRARY BOUNDARY CONDITIONS.

For a plate subjected to arbitrary boundary conditions at the edges the minimum critical buckling load is found by solving the eigenvalue problem represented by equation 3.34. This is a general form of the eigenvalue problem. The ma minimum eigenvalue of the matrix $[A] \pm [n]^{-1}$ [G] gives the minimum buckling load. This is determined by taking the inverse of the largest eigenvalue of the inverted matrix $(A)^{-1} \pm [G]^{-1}$. A general compertor program is made to evaluate the elements of matrices K, G and A^{-1} and to determine the largest eigenvalue of A^{-1} by an iterative procedure. The numerical values of the minimum buckling loads found by this procedure are given in Table 12 for various boundary conditions.

The variation of the minimum buckling load with the angle of orthotropicity is plotted in figure 9.

918 CONCLUSIONS.

The frequency of transverse vibration and the critical buckling loads of this rectangular generally orthotropic plates were evaluated using the haloigh Rits method and the principle of minumum potential energy. The deflections were assumed to be small and the effects of rotating inertia and shear deformation were neglected. In the vibration problem the frequencies are the eigenvalues of a real symmetric matrix. The natural frequencies of vibration for the first sixteen modes were calculated by using the Jacobi's method on the high speed electronic computer for various boundary conditions. But in the buckling problem the critical buckling loads were the eigenvalues of a real non-symmetric matrix (except when the pair of loaded edges are simply supported, in which case it is real and symmetric). The lowest critical buckling load is determined by an in iterative scheme.

tion and critical buckling loads is shown in the several tables and graphs presented. In some cases an orientation of orthotropicity other than 0° or 90° increases the minimum critical buckling load by as high as 100%. The advantage gained however will depend upon the side ratio, the boundary conditions at the edges and the orthotropic properties of the material. In applications where minimum weight is an important design consideration an investigation of this type is worthwhile in view of the improved buckling characteristics at certain angles of orthotropicity.

3.4 SCOPE OF FURTHER WORK.

The accuracy of the results of higher modes of vibration

could be improved by taking into account the effect of shear deformation and rotating inertia. The analysis could then be applied to thick plates also. Since plate like structural elements, of shapes other than rectangular, are frequently used in air craft and missile applications, it is profitable to extend the present analysis to such shapes (skew, trangular etc.). The effects of large deformations on the vibration characteristic could also be studied for the generally orthotropic plates. Further, the method of the present work could be extended to cylindrical and spherical shells to investigate the effects of angular orthotropicity on the vibration and buckling characteristics.

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APPENDIX AT

ELASTIC CONSTANTS FOR ORTHOTROPIC THIN PLATE

A.1 THE GENERALISED HOOKE'S LAW:

Robert Hooke in 1678 proposed his famous law, which in tensor notation can be expressed as

Tij * Cijke *ke

and Sij s Sijkl Tke __(1, j, k, l = 1, 2, 3) --- A.1

 $T_{i,j}$ and S_{kl} are components of stress and strain and $e_{i,jkl}$ and $s_{i,jkl}$ are the elastic stiffnesses and compliances respectively. There are, in general, S_i constants in each of equations $A_{i,l}$ but because of the symmetry relations existing between shear stresses and strains $(T_{i,j} * T_{j,l})$ and $S_{i,j} * S_{j,l}$, this number reduces to S_i . Hearmon (S_i) shows by a thermodynamic argument that $e_{i,jkl} * e_{kl,i,j}$ and $s_{i,jkl} * s_{kl,i,j}$. Thus the number of independent constants in each of the above equations reduces to S_i and the Generalised Hooke's law can be written as

$$T_{3} = b_{31}S_{1} + b_{32}S_{2} + b_{33}S_{3} + b_{34}S_{4} + b_{35}S_{5} + b_{36}S_{6}$$

$$T_{4} = b_{41}S_{1} + b_{42}S_{2} + b_{43}S_{3} + b_{44}S_{4} + b_{45}S_{5} + b_{46}S_{6}$$

$$T_{5} = b_{51}S_{1} + b_{52}S_{2} + b_{53}S_{3} + b_{54}S_{4} + b_{65}S_{5} + b_{66}S_{6}$$

$$T_{6} = b_{61}S_{1} + b_{62}S_{2} + b_{63}S_{3} + b_{64}S_{4} + b_{65}S_{5} + b_{66}S_{6}$$

$$A \cdot 3$$

There are thus 21 independent constants in equations A-3, because of the relation b_{ij} * b_{ji}. The stiffnesses and compliances relate the components of one second order tensor to those of another (equation A-1) and therefore form a fourth order tensor. They transform from one set of coordinate axes to another according the laws of coordinate transformation. Thus there are 81 transformation equations in 81 constants in the most general case which are reduced to 21 equations in 21 constants due to the relations cited above (5).

A.S THIN ORTHOTROPIC PLATE.

If a plate is orthotropic, i.e. it possesses three mutually perpendicular planes of elastic symmetry, and one such plane coincides with the plane of the plate, Scholmikoff (SO) has shown that the number of independent elastic constants reduces to 9 and the stress-strain relations become

In the case of thin plate, i.e. a plate whose lateral dimensions are large incomparison to its thickness, the state of stress is approximately plane and it can be assumed that $T_3 = T_4 - T_5 = 0$. Such a plate is known as a "generally orthotropic" plate and the Hooke's law in its case can be expressed as

If the two axes of elastic symmetry lying in the plane of the plate are parallel to the x_*y coordinate axes, then the plate is known as "specially orthotropic". Then $b_{16} * b_{26} * 0$ and the Hooke's law becomes

Aut

Te = beefe

It can easily seen that

If the plate is isotropic we have $E_1 = E_2 = E_3$, f(x) = f(x) = f(x) and f(x) = f(x), then

$$b_{11} = b_{22} = \frac{1}{(1-12)}, \quad b_{12} = \frac{3}{1-3}$$

and bee • 0

. A.8

the number of independent constants are then only two since a relation exists between E and G.

A.3 COORDINATE TRANSFORMATION.

For an erthotropic thin plate the elastic constants in any erhitrary directions inclined at an angle 8 to the axes of elastic symmetry in the plane of the plate are given by coordinate transformation (5)

where b_{11} , b_{22} etc. are elastic constants along arbitrary directions b_{11}^1 , b_{22}^2 etc. are elastic constants along axes of elastic symmetry n . Cose and n = Sin θ .

CEANORS STATE HAS LONG A MENTY

COMPUTER PROGRAM

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FREQUENCIES OF GENERALLY ORTHOTROPIC PLATES

THE PROGRAM CALCULATES THE ELEMENTS OF THE 16 x 16 MATRIX K. FOR DIFFERENT SIDS RATIOS AND ANGLES OF ORTHOTROPICITY. THE ELEMENALUES ARE DETERMINED BY JACOBI'S METHOD AND THE NON-DIMENSIONAL PREQUENCY PARAMETERS ARE PRINTED FOR THE FIRST 16 MODES OF VIBRATION AS OUT PUT.

DIMENSION FA(4,4), FB(4,4), FC(4,4), FD(4,4), FB(4,4), FF(4,4)

DIMENSION BUCK (4,4), A(4,4), D(16,16), EGEN(16), ALAM(16)

DIMENSION LFO F2 (4,4), LF1F2 (4,4), LF1F0(4,4), 7(16,16),B(6)

DIMENSION x (6), LF1F1 (4,4), LF2F1 (4,4), LF2F0(4,4), LF0F1

BF1F0(4,4), BF1F1(4,4), BF2F1(4,4), BF2F0(4,4), BF1F2(4,4)

BFOFE (4.4)

HEAL LEGES, LEAFS, LEAFO, LEAFI, LEEFI, LEFE, LEGES

MEAD INTEGRELS OF BEAM CHARACTERISTIC FUNCTIONS AS DATA

MEAD 333, (LF1F1 (M, N), Nel,4), Me 1,4)

BEAD 339, (LFIFO (M. N), Mal,4), Ma 1,4)

READ 333, (LF2F1(M.N), N= 1.4), M = 1.4)

HEAD 333, (LPEFO(N,N), Nal,4), Nal,4)

MRAD 333, (RF1FO(M,N), Nel-4), Me 1.4)

READ 333, (BPEF1(M,N), Mal,4) M = 1,4)

READ 330, (BPSPO(N,N), Nala) Na 1,4)

HRAD 383, (HF1F1(Man), Mal,4) Me 1,4)

READ 534, (X(M), M = 1,4)

READ 384, (B(N), No 1.4)

PRINT DATA FOR OUTPUT REGORD

PRINT 333, (IFIF1 (M,N), No 1,4), No 1,4)

PRINT 333, (LFIFO (M,N), No 1,4), No1,4)

PRINT 933, (RFIFO(M,N), Mel,4), Mel,4)

PRINT 333, (LF2FO(M,N), Nel,4), Mel,4)

PRINT 333, (BF1F0(M,N), Nel,4), Mel,4)

PRINT 333, (BF2F1(M,N), Nel,4, Me 1,4)

PRINT 333, (BF2F0(M,N), Hel,4), Me 1,4)

PRINT 333, (BF1F1(M,N), Nel,4) Mel,4)

PRINT 334 (B(M), M.1.4)

FORMAT (6F12.3)

FORMAT (4F16.8)

34 FORMAT (2 x, 1678.3)

22 PI.4.*ATAN(1.)

DO 111 Me 1.4

100 111 16 1,4

LFOF2 (M, S) = BF2FO(S, M)

LF1F2(M,N)=LF2F1(N,M)

BF1F2(M, N) = BF2F1(N, M)

LFOFI(M.W). LFIFO(N.W)

Brofl(M,N). Brifo(N,M)

B11, B22 ARE ORTHOTROPIC PROPERTIES OF THE MAPIN 5 PIX+ PIXWOOD PLATE (DATA)

D11.03.12

M22+1+00

B12m 0.1206

313e0e0

D00=0.0

383mO ,2637

THETA: -15.0

31 THETA THETA 16.0

```
T THRTA PI/180.
CECOS(T)
 S-SIN(T)
 Co # C**2
 So = S**2
C4 = C**4
 84 = 8**4
BAY - Blo-8. - B33
DIL. DES ARE ORTHOTROPIC PROPERTIES OF THE PLATE FOR
           ANGLE OF ORTHOTROPICITY THETA
D11_811*C4 + B22*84 +2.*BXY*C2*82
B22 - B22+C4 +B11+84 +8.*RXY+C2+82
D13 - (B22*82-B11*C2+ BXY*(C2+82))*C*8
D23 - (B22*C2-B11*82-BXY*(C2-S2))*C*8
D12 - (B11 + B22-4.*B33)*(C2*S2 + B12*(C4 -84)
D83 . (B11 + B22 - 2.*B12)*C2*82 + B83*(C2*82)**2
R IS SIDE RATIO * A/B
R . 0.0
R . R.O.5
DO 1 M . 1.4
DO 1 No 1.4
```

DO 16 J = 1:4

DO 16 1 = 1.4

IF (M.EQ.J) GO TO 11

BM & B(M)

XN = X(N)

BN = B(N)

DELIMI . O.O

```
T THETA PI/180.
CECOS(T)
SaSIN(T)
Co . C++2
S2 # 8992
C4 ± C**4
84 . 8**4
 BXY - Ble-6.*B83
 DIL. DES ARE ORTHOTROPIC PROPERTIES OF THE PLATE FOR
           ANGLE OF ORTHOTROPICITY THETA
D11_B11*C4 + B22*84 +2.*BXY*C2*82
B22 - B22*C4 +B11*S4 +2.*RXY*C2*S2
 D13 - (B22*82-B11*C2+ BXY*(C2-82))*C*8
123 - (122 *C2 - B11 *S2 - DX *(C2 + S2)) *C * S
 D12 - (B11 + B22-4.+B33) * C2*82 + B12*(C4 -84)
D83 * (B11 + B22 - 2.*B12)*C2*82 + B83*(C2-82)**8
 R IS SIDE RATIO - A/B
R . 0.0
 A . 840.5
DO 3 N # 1.4
DO 1 80 1.4
DO 10 J = 1.4
 00 18 1 . 1.4
BM # B(M)
IN a I(II)
 BH = B(B)
```

IF (M.EQ.J) GO TO

101ML = 0.0

G

```
T .THETA PI/180.
```

C_COS(T)

SeSIN(T)

Co . C**2

Sp # S002

C4 = Cen4

84 . 8444

BKY . B12+2.*B33

C D11, D22 ARE ORTHOTROPIC PROPERTIES OF THE PLATE FOR ANGLE OF ORTHOTROPICITY THETA

D11_B11*C4 + R22*S4 +2.*RXY*C2*S2

B22 . B22*C4 +B11*S4 +2,*MY*C2*52

D13 . (B22*82-B11*C2+ BXY*(C2-82))*C*8

D23 . (B22*C2-B11*82-BXY*(C2-S2))*C*8

D12 * (B11 + B22-4.*B38)*(C2*82 + B12*(C4 -84)

D03 . (B11 + B22 - 2.0312) *C2*82 + 393*(C2*82)**2

C R IS SIDE RATIO - A/B

R = 0.0

4 R . R.O.5

DO 1 M # 1,4

DO 1 No 1,4

DO 16 J = 1,4

DO 16 1 = 1,4

BM + B(N)

XN = X(N)

BR = B(B)

IF (M.EQ.J) 00 TO 11

DELMI . O.O

```
GO TO 12
```

10 DEIMI = 1.0

12 IF(N.EQ.J) 00 TO 11

DELNI . 0.0

00 TO 13

13 DELNJ z 1.0

13 DIMNIJ = DBIMI*DBINJ

CMI = LFIFO(M.I)

CIM = LFOF1(M.I)

DMI = LFIF1 (M. I)

BMI : LP2F1(M, I)

FMI = IFEFO(M.I)

BIM = LFIFE(M.I)

DIM . IMI

PIM . LFOF2 (M.I)

CRJ . BFIFO(N.J)

DAJ . MIPI (A.J)

BM . MP2F1 (A.J)

FMJ = EFEFO(N.J)

BJN = BF1F2(N, J)

CJH # BFOF1 (N,J)

DIN . DNJ

PIN . EFORE (N. J)

FA(I, J) = D11+NM++4+DIMNLJ

FB(I.J) - D12*(FIM*FMJ*FMI*FJHI)*R*R

FC(I,J) . DES+XN++4+N++4+DEMBIJ

PD(I.J) . 4.*D33*DIM*NUA*DJN*R*R

```
PE(I,J) = 2.*Dl3*(EMI*CJN+ BIM*CNJ)*R*R
      FE (I.J) = 2.*DE3*(CIM*ENJ* CMI*EJN)*R*R
       ABCD = PA (I,J) + FB(I,J) +FC(I,J) + PD(I,J) +FB(I,J)+ FF(I,J)
       A(I.J) # ABCD
      CONTINUE
 15
       L = 441/-4+1
       J4= J44
       J8 . J+8
       J12 # J + 12
 C
       D'S ARE THE BUSHEMTS OF THE MATRIX K
       D(L,J) = A(1,J)
       D(L,J4) zA(2,J)
       D(L,J8) =A(3,J)
       D(L_0J32) = A(4.J)
       CONTINUE
 10
       COMPLHUS
       PRINT 2. R. RX. THETA
       FORMAT (2X, 3P20.8)
  2
       FORMAT (2X, 16F8+1)
 21
       BLEMENTS OF MATRIX ARS NORMALIZED
 C
       DD \bullet D(1,1)
       DO 28 N # 1,16
       DO 23 N = 1,16
       D(M_*N) = D(M_*N)/DD
 23
       CALL JACOBI (16,Del,NR,Z)
       DO 999 3-1,16
       ALAM(N) = SQNT(D(N_n)/PI+44+2D)
000
       PRINTS, (ALAM(N), N ± 16)
       FORMAT (2X, 1678,3)
       CONTINE
1000
```

```
STOP
           END
SIBFIC
            JACOBI NODECK
            SUBROUTINE JACOBI(N. 4. JVEC, M. V)
            THIS SUB PROGRAM CALCULATES THE N EIGENVALUES.
C
            AND BIGENVECTORS OF A SYMMETRIC MATRIX BY A
            METHOD OF DIAGONALISATION.
            DIMENSION Q(16,16), V(16,16), X(16), IJ (16)
            DO 14 I . 1.W
10
            DO 14 J = 1, N
            IF (I-J) 12, 11, 12
            V(I,J) . 1.0
11
            00 TO 14
            V(I.J) # 0.0
12
            CONTINUS
14
            MEO
15
            MI ± N-1
17
             DO 30 I = I, MI
             X(X) = 0.0
             M . I . I
             DO 30 J + MJ H
             IF (E(I) + ABS(Q(I,J))) 20,20, 30
             X(I) = ABS (Q(I+J))
20
             III (I) : J
            CONTINUE
 30
            DO 70 I + 1, M
 40
             IF (I + 1) 60,60, 45
             IP (MAX + X(I) 60,70,70
 48
```

IF(R.III.3.5) GO TO 4

IF(THETA.II.90.) GO TO 31

```
XMAX - X(I)
          IP . I
          JP = IH (1)
70
          CONTINUE
          BP8I = 1.08-8
          IF(XMAX - EPSI) 1000, 1000, 148
148
          M = M + 1
          IF (Q(IP,IP) - Q(JP,JP) 150,151,151
          TANG = -2. = Q(IP.JP)/(ABS(Q(IP.IP)-Q(JP.JP))
150
                            +SQRT(Q(IP.IP)-
       1 Q(JP,JP))**2 + 4.*Q(IP,JP)**2))
          00 TO 160
          TANO = 2.*Q(IP_*JP)/(ABS(Q(IP_*IP)-Q(JP_*JP))*SQRT
161
                           (Q(IP,IP)
       1 - Q(JP,JP)**8*4.*Q(IP,JP)**8))
160
          COSH-1.0/SQRT(1.0+TANG++2)
          SIME MANG*COSH
          QII = Q(IP, IP)
          Q(IP, IP) = COSH+2+(QII+TANG+(2,+Q(IP,JP)+TANG+Q
                                            (JP,JP)))
           Q(JP,JP) + COSN++2+(Q(JP,JP) + TANG+(B+Q(IP,JP)+
                                     TANG*QII))
           Q(IP_{\bullet}JP) = 0.0
           IF(Q(IP,IP) + Q(JP,JP) 169,163,163
          TEMP . Q(IP, IP)
162
           Q(IP,IP) + Q(IP,IP)
           Q(JP,JP) + TEMP
           IF(SIME) 164,165,165
          TEMP . . COSH
164
           go TO 170
          THEP & -COSH
166
```

60

```
170
           COSN = ABS(SING)
           SIMB . TEMP
153
           DO 350 I = 1.MI
           IF (I - IP) 210, 350, 200
           IF (I- JP) 210, 350, 210
200
           IF(IH(I) - IP) 230,240,230
230
            IF(IIH(I) - JP) 350,240,350
230
            K = IH(I)
240
            TEMP \pm Q(I,K)
250
            Q(I.K) # 0.0
            MJ = I + 1
            X(I) = 0.0
            DO 320 J . MJ. H
            IF(X(I) - ABS(Q(I,J)))300,300,320
            X(I) = ABS(Q(I,J))
 300
             IH(I) = J
            COMPLINUE
 320
             Q(I.E) # TEMP
             CONTINUE
 360
             K(IP) ± 0.0
             X(JP) = 0.0
             DO 530 I . I.N
             IF (I - IF) 970,530,420
             TIMP # Q(I,IP)
  370
             Q(I,IP) . COSH-TEMP . SINE-Q(I,JP)
             IF(X(I)=ABS(Q(I,IP)))380,390,390
             X(I) = ABB(Q(I+IP))
  380
```

IH(I) = IP

```
390
       Q(I_*JP) = 8INE*TEMP + COSN*Q(I_*JP)
       IF(X(I)=ABS(Q(I,JP)))400,530,530
400
       X(I) = ABB(Q(I,JP))
       IH(I) m JP
       00 TO and 530
420
        IP(I - JP)430,530,480
       TEMP . Q(IP.I)
430
        Q(IP.I) = COSN*TEMP *SINB*Q(I.JP)
        TF(X(IP)=ABS(Q(IP,I)))440,450,460
440
        X(IP) = ABS(Q(IP_I))
        IH(IP) = I
        Q(I,JP) * +SINE+TEMP + COSN+Q(I,JP)
480
        IF(X(I)-ABS(Q(I,JP)))400,530,530
480
        TEMP = Q(IP.I)
        Q(IP,I) = COSN+TEMP +SIME+Q(JP,I)
        XF(X(IP)+AB8(Q(IP,I)))490,500,500
        X(IP) . ABS(Q(IP,I))
 490
        IH(IP) . I
        Q(JP,I) * *SIME*TEMP *COSN*Q(JP,I)
 500
        IF(X(JP)+ABS(Q(JP,I)))510,590,590
        K(JP) = ABS(Q(JP_*I))
 510
         IR(JP) . I
         CONTINUE
 530
         IP(JVEC) 840,40,540
         9, I 038 0G
 540
           I.IP) . Cosh-Thur . SING-V(I.JP)
           I,JP) . -SIMBOTEMP +COSHOT(I,JP)
 550
            TO 40
         ASTURN
1000
         TIMD
SENTRY
```


an	10 01 W W	8504	9 C 44 S	으크용옷
M. W. da	2,11,518 4,000 5,790 5,266	7.887	2,788 7,880 11,158	5,256 7,664 11,155 -164,868
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						806.8		31,905
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SHOUTHING HAM DESIGNATION OF STATES

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Fundamental frequencies $\left[\lambda * (fh/a^2b^2/T^4D_{126})^{1/2}\right]$ of specially exthetropic SSSS plates (Maple plywood) $D_{11} / D_{126} = 1.545$, $D_{22} / D_{126} = 4.815$)

Side I	****	0			90
a/b	From equation 2.20		by Haleigh's Method Hearmon (Ref. 5)	From equation 2.20	by Raleigh's 'Method Hearmon' (Ref. 5)
0.4	8.525	*	8,4528	5.885	5.686
0.6	2.852		2+852	5.991	3,991
0•8	2.737		2.757	5.242	5.242
3.40	2.891		2.891	2.891	2.891
1.2	3,165	, to	3.165	2.750	2.750
2.0	3+496		5+493	2.735	2.755
1.6	8.865		5.838	2.798	2.798
3.8	4.251	- '	4.251	2.913	2.915
2.0	4.652		4.652	5.062	5-058
2*2	5,081	,	5.031	8.285	3.236
2.4	5,476		5.476	5.434	5,424
2.6	5+805		5.896	8.625	5,625
2.0	G.ESt 519		6.519	8,055	5.655
5.0	6.745		64745	4,052	4,062
3.2	7,172	1000	76170	4-274	4.874
8-4	7.601		7.601	4,500	4,500
3,6	0.061		8,031	4.750	4,780
5.8	8,452		8.462	4,961	4,981
4.0	8,894	3	8,894	5.195	5,195

Natural frequencies $\left[\lambda = (9 \text{ b w}^2 \text{a}^2 \text{b}^2 / \text{ a}^4 \text{D}_{126})^{1/2}\right]$ orthotropic SSSS plate (Naple) plywood plate, D₁₁ / D₁₂₆ = 1.545; D₂₂/D₁₂₆ =4.815)

ido Milo	****			A	ale of o	REHOTROP	ICITY DE	GBS:88	
a/b	a	n	0	1.5	80	45	60	76	90
0.4	1	1	5.525	5,689	4.095	4.557	5.047	5,498	5,685
	2	1	12.770	12,810	15.462	15,568	18,440	21.076	22.126
	3	1	28.283	28,085	29.087	83.600	40.755	47.085	49.546
	4	1	50,016	49,422	50.845	59,191	72.278	85.568	89.958
	1	2	5.474	5.879	6.587	6.885	6,753	6.556	6.485
	1	8	9+488	9.762	10,216	10-150	9,461	8.617	8.251
L.O	1	1	2.891	8+022	5.292	3.427	3.292	5.022	2.861
	2	1	6.124	6.432	7.085	8.795	8.899	9,154	9,505
	3	1	12.158	12.707	12.239	18,628	18,490	19,506	20,284
	4	1	20,780	21,086	22.254	26.145	50,798	54.951	55.576
	1	2	9,505	9,154	8,899	7.555	7.055	6.432	6.124
	1	3	20,234	19,506	18,490	15.256	12,239	12,707	12.157
LaG	1	1	3.805	5.840	5.801	5,697	5.408	5.008	2.798
	2	1	5.474	5.755	6.561	6.835	0.886	6.629	6.485
	8	1	8,838	9,549	10,539	11.255	11.745	15,558	15,200
	4	1	14.098	14.641	15,005	20,079	21.745	22,542	22.745
	1	2	14.844	15.760	12,586	10,861	9,500	8,797	8,549
	1	3	51.885	80,408	26,859	22,468	19,564	16,524	18,435
	-	-	GTECON	es estimates		The same of the sa		4. 3. 20.	

side Ratio	4th min			Alkija	CO CONTROL	BOPKOLET I		Andrewson which the control of the con-	
a/b	13	3	0	15	30	4.5	60	75	90
2.0	1	1	4,652	4,548	4.833	4.010	5.673	5,258	5 .03 8
	2	1	5.781	5,975	6.436	6.740	8,609	6.087	5.781
	3	1	8.278	8.721	9.705	10,558	11.249	10,283	11-00
	4	1	12,248	12,916	14.865	16-275	16.651	10,186	18,80
	1	2	17.788	17.005	15.100	12.622	10,881	10,588	10,390
	1	3	59,722	87.788	88.908	27.738	23,674	22.715	32,75
8.0	1	1	6.745	6-480	5.834	5.154	4-470	4,194	4.CBS
	2	1	7.888	7.823	7,865	7.100	6.752	5.965	5.51
	3	1	0,072	8,951	9,500	10,077	9*9T6	9,141	0.07
	4	1	10*818	114535	15.090	14-178	14,822	18.055	19,6
	1	2	20,481	25-185	21,004	10,207	15.005	15.155	25.10
	1	8	59,507	56.887	40,045	39,900	34.450	85,485	55.61
0.0	1	1	0,004	8,498	7,480	6.500	5,619	8.207	8.19
	8	1	0,508	9,078	0,600	8,088	7,372	0.031	0.15
	8	1	10-140	10,176	10,828	10,427	0.944	6.020	8,10
	4	2	21,108	10,123	\$3.000	15.675	13,470	12,263	11.50
	1	2	50,217	55,459	86,604	25,702	80,555	10,904	80.08
	1	8	70,000	75.016	04,554	52,476	45,409	44.315	6. 8

TABLE S

Bunkling loads and Natural Proquencies of Specially orthotropic 8888 photos (Maple) plywood plate D_{11} / D_{120} = 1.545, D_{22} / D_{120} = 4.815)

			P ₂ (e II _e e	(D_)	
1	4	13	Male Retain O ₄ 5	Side Retio 1.0	Side Bubile 2.0
*8	1	1	9*005	9,615	15.500
	1	2	2.404	3.948	24.381
	2	1	48.807	49,475	57.888
-4	1.	1	5.002	5.615	11.500
	1	2	1.401	2.048	25.381
	2	1	58.507	58-475	41.886
0	1	1	1.000	1.615	7.800
Λ.	1	2	0.404	1.048	55 .581
	2	1	10.507	17.475	25.033
4	3	. 1	-2,900	+8.505	5.896
		. 2	+0.593	0.848	21-301
		1	0, 807	1.475	9,880
8	1	1	-0,900	+8.505	-0-867
			+1,693	-0*725	20,563
		1	*10,686	*4.4.505	-0,104

Material Engineery + λ = ($\frac{1}{2}$ = $\frac{1}{2}$ / $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

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Proguencies $\left[\lambda * (\text{ph } v^{2} a^{4} / v^{2})^{1/2}\right]$ of generally exchotropic cost plates (tople plywood, $D_{11} / D_{22} * 3.117$, $D_{33} / D_{22} * .262$, $D_{12} / D_{32} * 0.12$)

aldo		8				80	75	90
2010.			3	80			10	
0.5	1	4.104	3.962	5.522	5.CE	2,687	2,579	2.570
	2	4,509	4-407	4.529	4.000	8.778	8.785	5.625
	8	5.487	5.529	5.000	5,520	5,280	5,618	6-199
	4	6.974	7.100	7.508	7.560	0.584	6.467	6-471
	S	11,151	10,617	9,296	8,005	7.801	7.061	7.823
	0	11,482	11-110	10,255	9,100	6,271	8.148	0,154
1.0	1	4,013	4.745	4,800	4-545	4,600	4.743	4,813
	2	7.907	7.905	8,255	8,490	0,955	7.906	7,907
	8	11,500	11,000	10,379	9,077	10,879	11-808	11,589
	4	18-671	15,599	18,280	15,007	18,266	15,800	10.491
	5	15,720	14,077	14,007	15,87	14,867	14,077	15,780
	6	10,000	18,547	10,057	17,280	18,067	18,547	10,220
1.5	3	0,001	0.019	7,008	7,597	8,470	0.231	0.527
	8	12,000	12,818	11.717	11.845	11,905	22.700	11.70
	8	10,170	15.300	10,100	17,818	17.478	10,742	10,000
	4	10,500	10,007	17,000	10,324		25,575	25,521
	5	20.CL0	22.620	28,070	25,050	25,248	24,503	
	G	20,300	25.700	90.440	20,037	35-535	26.507	20,79
2.0		10,800	10,819	10,740	12,413	14,007	15,709	15,42
		15.000	14,010	15,001	10,855	17,614	17,000	18,05
		24.705	62,471	81.043	m.000	22,650	2 ,W	22.70
		05,005	65.050	20.755	80,629	80,891	20,729	27.64
	4	20.200	80,644	82420	80,019	57,194	42,400	44,02
	5	23,530	82,894	89.004	50400	40,941	44,464	43,6
	0		44.54		and the same of the same		e national transfer en	

MAG Prodo Vo	Modo-	0	25		45	60	75	90
2.5	2	15,137	n'm	15.780	10,000	21.578	24.250	25.389
	2	19,214	19,051	19,825	22.156	24.478	26,106	20.678
	3	27.736	25,656	25.148	27 .81 8	29.615	29,780	29,824
	4	89.002	30.905	38,501	37.157	87,210	35.078	54,905
	8	40,657	39,885	41.858	47,519	57.505	65.989	09,541
	6	42.607	43-679	45.007	55-056	61.145	67.898	70.491
8.0	1	21-214	21.108	21.036	25.255	50,502	84,861	56,348
	2	24.600	24,570	25.012	29.388	\$5,883	56.355	57-474
	5	32.084	30,295	80-811	55.006	88,386	89,654	89,965
	4	44,111	41.903	41,498	44,594	44.765	44,920	44.288
	5	56.988	56.471	58,811	67,471	82.094	94,744	100,759
	6	69,442	60,172	05,025	75.106	96.986	90,000	100.700
8.8	1	20,482	20,286	29,228	85,795	40,000	46,974	49,541
	2	S1.572	81.584	53,234	87,979	45,049	48,580	50,358
	3	37.010	80.400	57,984	45,742	40,000	53,003	58-489
	4	48,000	47,063	40,000	58,555	60,000	60,089	60,020
	5	77,220	70,368	70,505	81,000	773*789	2:0-763	135,545
	0	79,400	70.609	01,104	90,770	114,000	1804080	186,570
			714		***			

Frequencies $\left[\lambda = (\text{ph } v^2 a^4 / \pi^4 D_{22})^{-1/2}\right]$ of generally exhibitropic SSCC plate (Maple plywood $D_{11} / D_{22} = 5.117$, $D_{66}/D_{22} = 262$; $D_{12} / D_{22} = 0.12$)

ado	Mode	AMGA	e of ortho	Taopiciti	(DECERTES)			
Bertlo e/o	No.	0	15	80	45	60	78	90
0.5	1	1,80	1.968	1,900	1.800	1,669	1.578	1.550
	2	2.657	2.791	5.CSO	5,111	2.089	3.C07	5.181
	3	8.989	4.208	4.619	4.705	4.517	4.378	4.510
	4	5.084	6.141	6.156	5.280	4.654	4.805	5.411
	5	7,198	6.192	6.754	6,070	6.433	6.500	5.789
	6	7,634	7.590	7.587	7.048	0.809	7.012	7.635
1.0	1	3+141	5.229	8.462	5.758	4.028	4.252	4.518
	2	6.948	7.000	7.048	6.910	6,651	0.545	8.200
	8	7,035	7.884	8-184	8.910	0.953	10,744	10,522
	4	10,829	10,909	11.843	21,451	11,175	10,988	10,852
	5	12.891	15,000	13.941	15,031	15.057	12,784	12,730
	6	15+877	15,657	14.972	16,200	16.614	18,888	16.029
1.5	1	5.925	5,260	6,250	7.079	8,108	0.941	9,288
7.00	2	9,504	9,481.	9.745	10,508	10,640	10,881	10,567
		14,648	14,788	15.195	15,550	15,141	14,598	13.851
	4	17,393	15,958	15,081	17,957	21,148	20,516	19,000
	5	17,688	19,094	20,709	22,500	22.088	24,050	25,127
	6	25.594	28,229	28.142	28,096	20+858	25.839	26.215
			9,645	10-214	11.750	w.e.7	15.872	10,240
5.0	1	9,501		15,520	15.000	16,324	17.053	17.272
	8	12,565	12.000	10,640	20,052	80,668	20.257	10.00
	3	10,807	18,703	28.517	20.102	27,410	25,409	24.801
	4	25.525	25-43.0	20,154	80.060	30,968	40.540	44,454
	8	27.771 81.889	28,857	81.548	35,018	40,108	65,987	45.40

Sido	Mode	ARRIVA	OUTOR DOWN	PECITY				
Rittle VA	1804	•	16	5 0	45	60	75	90
2.5	1	14.027	14.621	15.325	17,695	21,153	24,105	25.246
	2	17,055	17.808	18,595	21.08	25,000	25.478	26.127
	3	25.315	22.557	25,409	26,139	27.040	28,510	26,247
	4	34-058	36.98	52.900	54.521	54-544	83.185	32.295
	5	39.576	39.384	40.778	47.188	57.170	65.00L	69,265
	Ø	41.547	42,404	45,608	52.054	60,400	87.487	70.185
3.0	1	20.850	20.728	21,589	24.968	50,111	54,538	55.252
2	2	22.004	25.195	24.057	20.452	52.826	55+948	57.051
	3	28,266	27.850 29.548 55.618 56.672		58-442	38.832		
	4	4 39-014 37-81		59,148	42.144	45,502	42.700	42.266
	5	56,748	56,186	57.928	07.169	81.051	94.647	100,507
	6	58,568	59-102	62.818	72,189	85*500	96,943	108-188
8.5	1	28-179	27,959	59*979	85.557	40,694	46.869	49,962
	2	30,000	30,298	52,294	57-118	48,817	49-140	50, OL1
	8	84,057	34-516	50.977	42,421	47.450	50-575	51.622
	4	48-404	45,715	46.551	51.050	55.040	54.500	54,66
	5	77,080	76,181	78,255	90,809	111*005	128-840	
	8	70.776	79,025	05,153	85,884	114,595	150,251	136.55

Frequencies $\lambda = (pin^2e^4/\sqrt{4} p_{22}^2)^{1/2}$ of generally esthetropic sess place (maple plywood, $p_{11}/p_{22} = 8.117$, $p_{80}/p_{22} = 262$, $p_{12}/p_{32} = 0.12$)

a stillible arrived, dec.	Mode		A	iole de o	CIOTRIPI		388	
A A	llo•	0	15	50	45	60	78	90
0.5	1	1.078	1.842	1.745	1.581	1.579	1.250	1.282
	2	2.327	2.454	2,600	2.580	2.347	2-250	2.527
	3	8-881	3.554	5-911	8-959	5.780	4.009	4-165
	4	4-950	5.225	5.824	5-161	4.485	4.017	4-440
	5	7.161	6.633	0+100	6.830	5.805	5.259	4,260
	6	7-490	7.880	9,000	6,500	6.673	6.581	6.500
1.0	1	2.327	2,553	2,472	27523	2,477	2,357	2,327
	2	4-960	5,018	5,293	5.659	5.659	5,174	4.950
	3	7-490	7,883	7.075	6.053	6-841	7-252	7,490
	4	9,309	9,087	9,210	9,651	9,908	9,009	9,509
	8	9,707	10,300	11,100	12,017	11,126	10,070	9,787
	6	13,300	15,500	25,000	15,180	14,007	14,116	15,300
1.6	1	3,831	3,380	5,723	4,145	4.872	4,435	4-440
	2	0,104	7.549	7,201	7,654	7,748	7,020	6.45(3)
	8	9,767	10/187	10,957	12,800	10,000	17*507	11+000
	4	18,398	12,401	11.700	/20,770	14,130	15,578	10-1191
	8	10,053	12,057	17,004	18,004	17,907	17,671	17,759
	8	20,945	19,081	18,807	20,548	20,245	19-604	17,845
		4-930	5.008	5,520	0,320	0.00	7,843	7,490
2.0		9,509	0,737	6,000	10,299	10,553	8,707	
	8	10,710	14-407	15,204	15,610	\$5,904	14,380	15,500
	8	17,759	10.773	17,608	20-420	25,859	57*007	19,720
	4		23,041	22,005	24,576	24,427	27,427	20,844
	8	19,720		mulcidit.		28,545	23,574	29433
	0	20,509	The contract of the contract o	105 To 100				

ratio	Do	Ö	The literature of the later of					
	A strain the land on the		45	50	40	60	78	90
2.5	1	7.091	7.189	7.844	9,00	10,287	11,141	11.444
	2	11+017	10,507	11,219	15.574	14,024	15.473	15,049
	3	19,007	15,92	15,433	19,659	19,610	17.808	16,607
	4	25.701	24,691	24.710	28,151	27,727	24.333	22.527
	5	28,334	27,005	28,833	31.601	37 . 204	42.507	44,585
	8 ,	51,108	30,167	55.490	37.545	41-487	44.650	45.778
5.0	1	0.4007	9,000	10,657	12.305	14,200	15.750	18,201
	2	33 ₀ 393	15,070	14,240	17,022	18,104	18,001	17,759
	5	20,045	17,088	18.858	22,048	24,002	22,194	20,945
	4.	32.055	28,300	29,415	35,094	35,4000	26,517	26,870
	5.	30,000	86+013	58,500	44-101	55 ₀ ,120	60,981	65,822
	6	39,148	40,708	44,500	50,421	57,450	65,061	65,162
8.5	1	18,000	18,010	15.900	10,200	10,908	21,108	22,024
	2	10,000	10,025	17,882	87*877	25.020	25,495	28,405
	3	25,360	20,730	22,074	27-429	29,005	27-471	28.507
	4	84,682	58,202	55,908	55,520	57,705	38,608	81,277
	8	49,678	49,400	51,855	89,805	71.000	60,897	86.025
	6	50,000	55,655	87,030	05,787	70-197	64.634	88+097
4.0	1	10,729	16,745	17,700	20,658	24,404	27,475	29,644
	2	10,720	10,000	22,023	25-028	25,538	29,678	29,061
	5	00,809	24,000	20,440	22.53	34,790	85,627	52. 4 555
	4	67,236	85,000	58,777	45,905	45,780	50,589	57,280
	5	04,009	64,185	66,304	70,733	95,500	107-308	135,417
	0	00.010	67,563	72,600	65,4550	07,800	100,825	114,570

Proguencies \[\lambda = \lambda \frac{pin^2 a^4}{\pi} \frac{a}{\pi} \frac{1}{\pi} \frac{1}{2} \quad \frac{a}{\pi} \frac{1}{2} \quad \frac{a}{\pi} \quad \frac{a}{\pi} \quad \frac{1}{2} \quad \frac{a}{\pi} \quad \frac{a

aldo	Mode		A	iole or or	POTROPICE			
older Older	llos	0	35	80	45	60	75	90
0.5	1	1.789	1.710	1.507	1.271	1.105	1,015	1.050
	2	1,985	2.020	2.087	2.035	1.019	1.535	1.400
	8	2.622	2.005	3,100	4,685	5,059	2+655	2.857
	4.	3.788	4.107	4.000	5,289	4.101	8,990	4.041
	5	7.008	6,741	5-848	5-135	4,984	4.585	4.894
	6	7-270	7.004	6.489	5,784	5-114	4,884	5.192
1.0	2	1.502	1.790	1,654	1,510	1,270	1,214	1.145
	2	2,605	2.050	5,211	5,447	3,591	2.005	2,066
	3	6-116	6,215	6,927	5,170	4,476	4-100	4-105
	4	7-156	0.034	6.851	7,010	6*378	6.152	5,607
	5	7.911	0,000	8-400	8,808	9,071	9-105	9,159
	6	10,490	10,735	11,129	10,897	9,560	9,580	9,467
1.5	1	1.954	1.001	1,756	1.603	1.772	1,455	1,507
	2	3-074	3,027	8.955	4,350	4,682	6,675	4,535
	8	7,202	2,017	6,000	6-456	5.005	5,121	4,700
	4	9,000	0.000	8,104	9,600	9,309	8,737	7,701
	S	12,303	10,741	13,157		10,894	9,420	9,545
	6	10,010	14,285	33,767	14,604	15,513	14,050	12,616
2.0	1	2,024	1,052	1.602	2,078	2,118	1,740	1,506
	2	5,550	4,585	8,483	4,701	5,000	4,672	450
	8	7+408	7,581	0,000	9,500	7,508	6,00	1,897
	٥	10,783	0,000	7,008	6+80t	16/181	9#842	9.600
	5	16.250	15.688	14.516	15.807	14.391	11.976	15.664
	6	19.469	16.610	16.701	16.379	16.997	16.687	15.539

retio	Mode No		and the state of t	AMOLE OF	ORTINE	PICITY		
1	Alfahari Mariatanan	Ö	15	30	45	80	78	90
2.5	1	2.255	1.658		24289	2,583	2.061	1.728
	2	7.727	4,444	1.672	4.071	5.974	5.558	4,875
	3	7.594	7,661	809	8,474	10,802	10,368	9,024
	4	12,000	8,711	8,588	10.971	11,789	11,785	11,636
	5	16-459	18,211	14,885	15,595	17,359	15.985	14,498
	6	°1.563	18-212	20,159	20,351	18,545	17,274	16.941
3.0	1	2.442			2,359	2,955	2.370	1.967
	2	7.817	2,347		4.905	6.714	5.854	5,220
	8	10,566	8,000	3.4070	8,013	11.725	10,974	10,509
	4	15,498	9,987	9*1CI	13,245	15,656	16,222	16,542
	5	10.000	16,502	16,434	17.831	19,017	17,953	17,540
	6	24-149	80*373	24-004	24,841	25,208	20,621	18,165
3.5	1	2,646			1,760	5+406	2,712	2,216
and also and	2	8,078			4,649	7.490	6.438	5,611
	5	13,682	1.005		7,500	12,061	11,654	10,748
	4	16,030	10,975	0,000	14,851	10,421	18,675	17,804
		18,615	17,590	19,078	20,500	20,920	21,519	21,804
	8	27,220		20,372			25,659	24.757

Frequencies $\left[\lambda = (pin^2 n^4/a^4 D_{22})^{1/2}\right]$ of generally orthotropic GFF plate (Maple plywood, $D_{11}/D_{22} = 3.117$, $D_{32}/D_{22} = .332$, $D_{11}/D_{22} = 0.12$)

aldo Mode ANOTE OF DESTROYAUT OVE ratio Non T I 80 45 9/9 76 100 60 0.5 1 0.629 0.590 0,505 0.410 0.355 0.349 0.356 2 0.781 0.756 0.775 0.726 0.588 0.500 0.518 3 1,155 1.840 1.385 1.555 1.174 1-183 1.200 5*00T A. 2,500 2-124 2.354 2.196 2.195 2.233 5 3-948 3.715 5.175 2.597 2.315 2.384 2,486 5,059 8 4-053 5,618 5.257 2.000 2.000 2.037 0.356 0.629 0.888 0.850 0.848 100 1 0,868 0,460 0,768 2 0.900 1.055 1-177 1,020 1.098 0.667 2,250 2,010 2.178 2.740 2,661 2,580 2,140 5 2.990 5-114 5,945 34608 3-194 5.250 5-240 4 4,265 4-281 4,380 4-800 4.557 8.394 4-400 8 6,105 6,078 6.770 6.000 0.215 6.400 0.147 0 0.347 0,556 0.344 0.381 0.629 0.400 0.351 1 1.65 1.081 1,259 1.279 1,000 1.557 1,500 1,059 2 2.170 2.250 **24**331 2.142 5,408 2,072 3.050 3 3.705 4,411 5,000 4-217 4.700 4-904 4,690 6-115 6.245 5,075 6.000 0.125 5,678 5,523 6 7.OLL 6,549 7.040 8,111 7.021 8,850 0.277 0 0-356 0,548 0.205 0.845 0.276 0.029 1 2.0 1,556 1-610 1-961 1-227 1.523 1-091 1.700 2.226 2.170 2,250 2,514 2,976 3,910 -4.474 5,670 5.228 2.005 4.007 4,625 5.597 6.245 6.00 6,159 6+904 7.608 0+477 9-450 5 6.009 0.045 11-017 9,733 9,805 9,479 11-041

alde Patlo	Mode No.	AND COLORED BOOK CONTRACTOR OF THE PROPERTY OF									
1/ 5		0	15	\$ 0				80			
?.•5	4	0.809	***	***	***	0,325	0,844	0.356			
	2	1.791	***	***	***	1.968	1.055	1,614			
	3	8,940	1.000	***	1.019	2.651	2.204	2.230			
	4	8.317	5.287	2.197	2.656	5.565	5.080	5.270			
	5	11.029	7.718	5.221	7.910	8,250	6,581	6.245			
	8	18.517	11.874	17*500	10,505	11,260	11.245	9,000			
8.0	1	0*889	***	***	***	0,308	0.848	0.556			
	2	2.086	***	***	***	1,000	5-101	1.835			
	3	3,960	***	***	***	3+138	2,414	2+250			
	4	7.074	9,4300	1.400	5,000	5.544	5.954	6,000			
	5	11,061	6,008	5,237	6,850	9,660	7.567	6.245			
	6	14,489	12.176	12.662	12,000	11,179	11,655	11.223			
3.5	1	0,689	***	***	650	0+288	0,840	0,886			
	2	2,542	***	***	***	1,702	5*110	2.175			
	8	9,940	***	***	***	8,630	2.702	2,230			
	4	7.002	1,751	0.011	2,617	5.001	5-022	0.244			
	5	11,081	5,905	5,810	C	10,577	8,842	6,906			
	6	15,500	15.008	34,238	15,755	11,505	TYS	12,257			
			20.								

TABLE 7

Frequencies $\left[\lambda_{*}(phw^2a^4/\sqrt{p_{22}})^{1/2}\right]$ of specially orthotropic plates (Maple plywood $D_{11}/D_{22} = 3.117$; $D_{12}/D_{22} = .12$ $D_{66}/D_{22} = .262$)

Angle	8100		Charles and Anti-Anti-Anti-Anti-Anti-Anti-Anti-Anti-				
of ortho- tropicity e degrees	Ratio a/b	From Eqn. 2.31	Naleigh's Nethod	From Eqn. 2.31	Ay Aeleigh's Method		By Releigh's Method
0	0.5	4.304	4.334	1.050	1.959	1.873	1.871
	1.0	4.813	4.817	3.141	3.140	2,327	2.325
	1.5	6.881	6.827	5.723	5.782	3,331	3.328
	2.0	10.306	10.321	9.581	9.519	4.930	4.926
	2.5	15.197	15.170	14.627	20.005	7.001	7.087
	3.0	21.214	81.269	20,830	80.898	9.787	9.783
	3.5	28,482	28,864	28,179	28.176	13.008	12.998
90	0.5	2.876	2,580	1.650	1.550	1,232	1.232
	1.0	4,813	4.817	4,318	4.316	2,327	2.325
	1.5	9.527	9.548	9.268	9.257	4,440	4,436
	8.0	16,417	16,457	16.948	16.230	7.400	7.485
	2.5	25.369	25,437	25,246	25.233	11,444	11.437
	3.0	36.348	36,451	36,262	36 .236	10.201	16.281
	3.5	49.341	49.484	40,262	49.236	22,024	85.078
						12 - 1	

Frequencies [\lambda = (phw2a4/ m4 D22) 1/2] of specially orthotropic cantilever (CFFF) plates (Maple plywood D_{11}/D_{22} = 3.117; D_{12}/D_{22} = .12; D_{66}/D_{22} = .26)

Side ratio a/b	Mode No.	from equation 2.31	from Reference 9
0.5	1	0.629	0.629
	2	0.731	0.730
	3	1.133	1.127
	4	2.001	2.000
	5	3.943	
1.0	1	0.629	0.689
	8	0.966	0.966
	3	2.740	2.740
	4	3,945	3.942
	6	4.408	
2.0	1	0*659	0.689
	8	1,239	1.195
	3	3*939	3,938
	4	4,944	
	6	5.523	5.601

TABLE 9

Buckling loads ($k_b = N_1 b^2/h^3 E_L$) of generally orthotropic SSSS plates (Mahogony plywood $D_{13}^*/D_{22} = 3.04$, $D_{126}^*/D_{22} = 0.438$, $E_L = 1.35 \times 10^6$ p.s.i.)

Side Ratio	103	o n	ANOME OF ORTHOPROPROMY, 6								
			0	J	30	45	60	75	90		
0.4	1	1	4.408	4.000	3,431	2.806	2.269	1.833	1.670		
	2	1	36,924	16.414	11,406	8.004	5.001	5.619	5.70		
	Э	1	37.824	35.886	31,287	16.352	12,321	12.043	12.54		
1.0	1	1	1.080	1.209	1.494	1.645	1,494	1.209	1.080		
	2	1	5.983	5*558	2.690	2.650	1,688	1.383	1.236		
	3	1	6.237	5.764	4.822	3,463	2.284	2.064	2.240		
1.6	1	2	1.015	1.200	1.508	7*805	1.000	1.075	1.990		
	8	2	1.377	1,468	7*658	1.618	1.384	1.090	1.963		
	3	3	2,608	2,000	8.693	1.001	1.023	1,294	1.163		
2.0	1	2	1.236	1,420	1.843	2.234	2.539	2,789	2.929		
	8	1	1.080	1,233	1.515	1.587	1.405	1.174	1.080		
	3	1	1.795	1.780	3.600	2,72	1.420	3.115	0.985		
2,4	3	1	1.071	1,740	2.174	8,679	3,020	3,812	4.08		
	2	1	0,972	7*703	1,408	1,031	1.638	1,962	1.307		
	3	1	1.977	1.470	1.653	1,612	1,803	1.079	0.96		
3.2	1	1	2,503	2,618	9.053	3.885	6.002	6,453	7.06		
	8	1	1.018	1,211	1.619	1,869	1,600	1.953	1.99		
	3	2	1.020	1,197	1,604	1.585	1,420	1,218	1.14		

TABLE 10

Normal $(K_b = N_1b^2/h^3E_L)$ and Shear $(K_5 = N_12b^2/h^3E_L)$ Buckling loads of square specially orthotropic SSSS plates (Mahogomy plywood $D_{11}^4/D_{22}^2 = 3.04$, $D_{126}^4/D_{22}^2 = 0.438$, $E_L = 1.35 \times 10^6$ p.s.i

Angle of orthotro-	NORMA		O-EFF.	SEPARTELISMANICE COSTAT.			
picity degrees	From Eqn. 2.20	From Reference	*	From equation 2.20	From Poference 16		
0	1.080	1.08	and the second s	2,566	2.456		
				-2.566	-2.56		
15	1.209	1.21		2,241	2.24		
				-3.607	-3,61		
30	1,494	1.60		2.399	8.41		
				-4.900	4.80		
45	1.645	1.65		8.625	2.53		
				+5.511	-5.50		
60	1,494	1.50		2.309	8.41		
				-4.900	~4.29		
75	1.209	1.21		2,941	2.94		
				-3.607	+3.61		
90	1,080	1.08		2.566	2,56		
				-2.536	-B.56		

TABLE 11

Shear Buckling loads $K_8 = \frac{N_1 2 b^2}{h^3 N_L}$ of generally orthotropic SSSS plates (Mahogomy plywood $D_{13}^4/D_{22} = 3.04$, $D_{126}^4/D_{22} = 0.438$, $E_L = 1.35 \times 10^6$ p.s.i.)

ingle of	SHEAR	BOUNTAING CORFFIG	Tems K
olcity legrees	a/b = 0.5	e/b = 1.0	a/b = 1.5
0	-9.731	-2.56 6	-1.670
	9.731	2.566	1.679
15	-13.843	-3. 607	-2.127
	7.778	2,241	1.504
30	-16.302	-4.900	-3.130
	6.951	2,399	1.794
45	-13,805	-5.511	-3,966
	6.656	2,525	1.848
60	-10-200	+4.900	-4.046
	6.671	2,399	1.834
75	-7.000	+3,607	-3,504
	6.038	2,941	1,980
90	-6. 795	+2,566	-2,464
	6.735	2,566	2,464

FARIE 12 Bucling loads $(K_b = N_1 b^2/\pi^2 D_{22})$ of generally orthotropic plates with various boundary conditions (Haple plywood D_{11}/D_{22} = 3.117, D_{12}/D_{22} = 0.12, D_{66}/D_{32} = .26)

loun- dary	Side ratio			AND E TOP	ORAHOIR	0)2109344		
	- a/b	0	16		45	60	75	90
	0.5	52.571	48,403	30,429	28.247	22,181	20.542	20,621
	1.0	17.915	17.066	15.660	14,925	14.767	34.725	14,715
	1.5	14.967	14.085	13.173	13.340	13.049	12.047	11.492
C	2.0	13.787	11.788	10.588	11.816.	12,545	12.010	11.569
ເຼັດ	2.5	11,819	10,048	9.717	12.084	13,794	13.992	13.837
~	3.0	11.165	9.894	10,164	13.339	16.095	17.235	17,480
	3.5	11,519	10.546	11.380	16.288	19.163	21.453	22,195
	4.0	12,566	11,875	13,101	17,733	22.879	26.522	27.861
	0.8	15.350	15.153	14.344	12,775	10.990	9.760	
	2.0	9+866	10,351	11.750	12,135	10.899	10.195	***
	1.8	10.126	30.763	12.287	11.734	11.444	10.256	***
8 8	8.0	9.822	9.794	9.501	11,099	11,868	11.398	***
C	2+8	9.561	8,925	9,608	12.079	13,658	14,247	***
	3.0	9.891	9,451	10,698	19,674	16.768	18,242	***
	3.5	10.862	10.711	12,268	16,249	20.652	23.199	
	0.5	34,026	13,570	12,185	9,997	7.602	6.247	6.076
	1.0	6.436	5.560	6+113	6,617	6.134	5.668	5,416
8 8		4,933	5,105	6.360	74629	6,432	5,123	4.825
8	8.0	5. 416	6.120	6+342	6,934	6*818	5,843	4,93
	2.5	4,864	5.689	4,621	7+102	7,453	6,121	4,90
	3.0	4. 932	5,381	6+481	7,213	6,688	5.932	5,41
	3.5	9.948	5.231	6,392	7,814	6.623	5.821	***

Boun- dary	Side Patio			E 07 0 80	Horaopio	TTY		
condi- tions	e/b	0	. 13	30	46	GO.	76	80
	0.5	12.807	11.695	9,069	6,447	4.878	4.365	-
	1.0	3,431	3,180	2,679	2,261	1.660	****	***
971	1.5	1.698	1.487	1.227	1.239	1.308	0.940	***
F S	2.0	1.093	0.768	0.428	0.876	1.090	0.755	
	2.5	0.814	.321	0.099	0.888	0.879	0.670	***
	3.0	0.662		0.165	0.288	0.907	0.684	
	3.5	0.572	0.206	-	0.075	-266	0.594	
	0.5	3.122	2.722	1,967	1.307	0.983	0.957	1.000
	2.0	0.780	0.599	0.391	0.280	0.238	0.238	0.250
_	1.5	0.347	0.188	0.078	0.086	0.201	0.105	0.110
C F	2.0	0.195	0.027	***	0.015	0.054	0.058	0.068
y	2.5	0.125			****	0.038	0.037	0.040
	3.0	0.087	-			0.020	0.025	0.028
	3.5	0.064	-	.157	-	0.033	0.018	0.020

Buckling loads ($S_D = N_1 b^2 / \pi^2 D_{22}$) of specially orthotropic plates (Maple Plywood D_{11}/D_{22} 3.117, D_{12}/D_{22} = .12; D_{66}/D_{22} = .26)

Angle of	8100	COOL		883		988	
Orthotro- picity Degrees	ratio a/b	From Equn. 3.34	From Bof.16	From Equa. 3.34	From Ref. 16	From Eqn. 3.34	From Ref. 16
0	0.5	52.571	52.64 8	15,350	16.641	14.036	14.020
	1.0	17.915	18,208	9,86	10.181	5.416	5.416
	1.5	34.967	15,499	10,126	10.275	4.933	4.93
	2.0	13.787	12,391	10.822	10.181	5.416	5.436
	8.6	11.815	11.850	9.661	9.925	4.854	4.856
	3.0	11.165	11,949	9,891	10,181	4.932	4.933
	3.5	11,519	11,725	10,862	11,280	4.948	4.940

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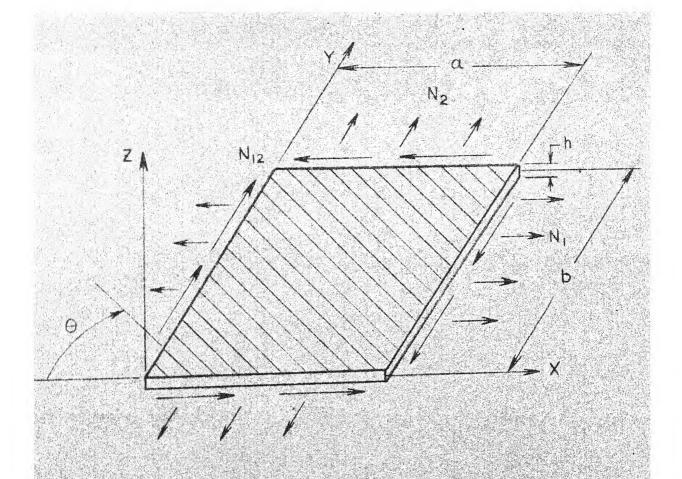
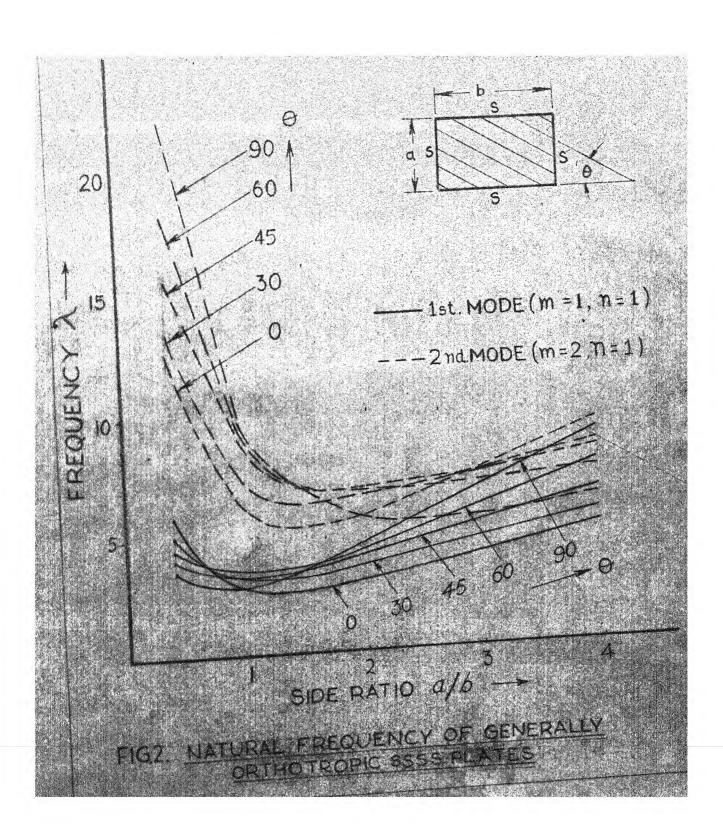


FIG.1 PLATE GEOMETRY



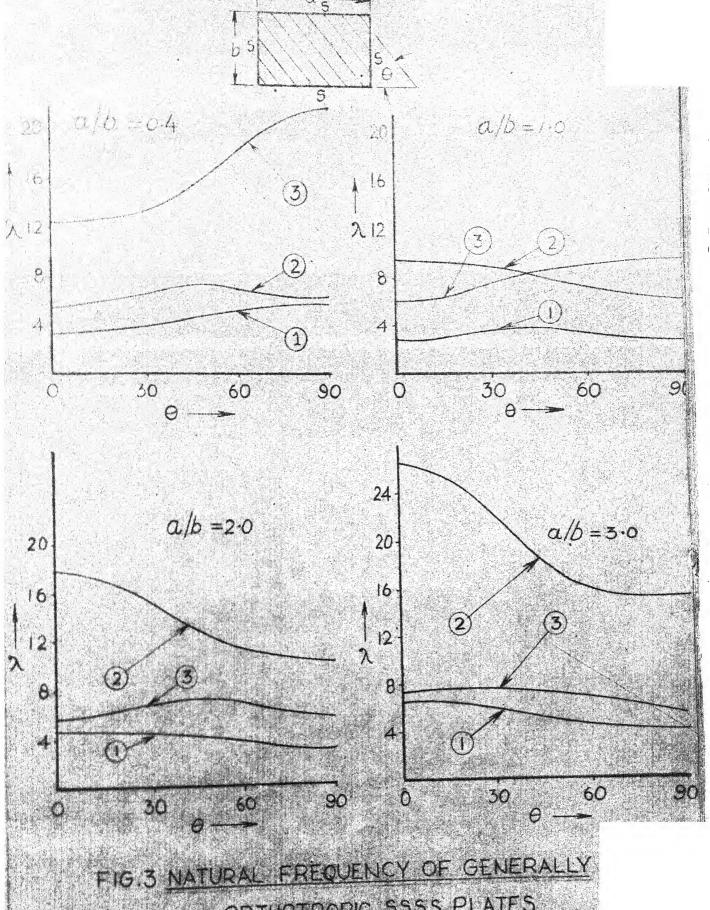
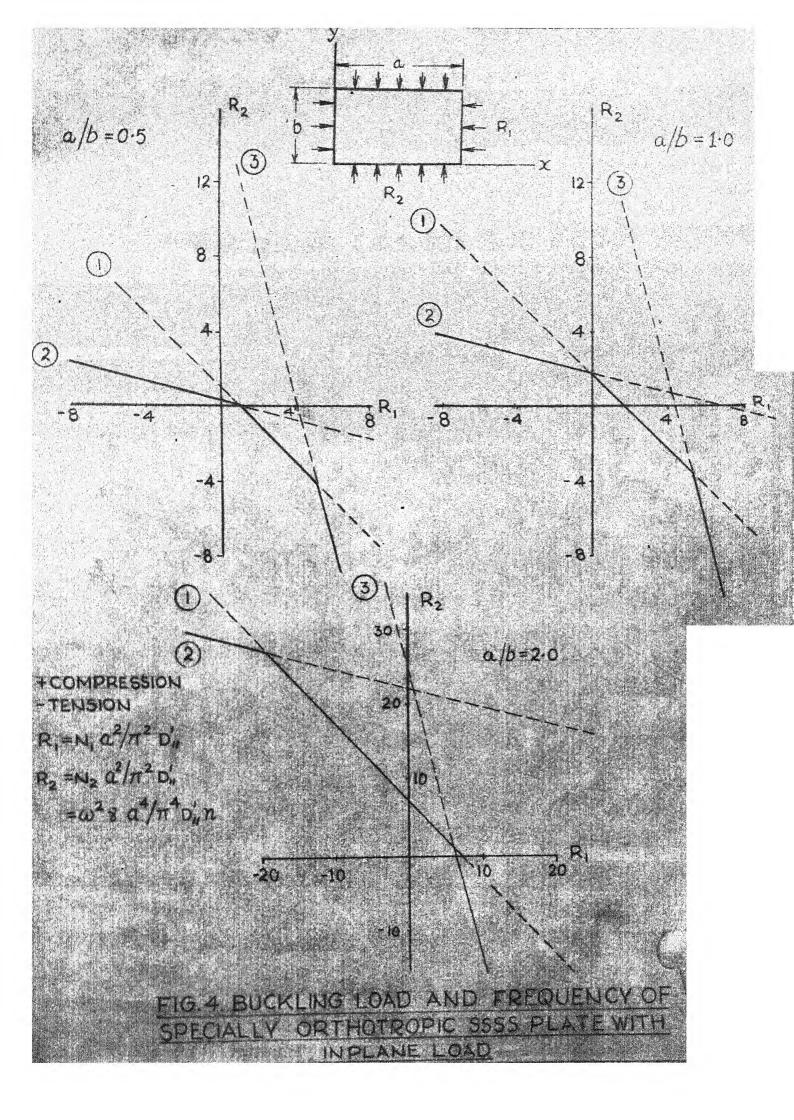


FIG.3 NATURAL FREQUENCY OF GENERALLY

ORTHOTROPIC SSSS PLATES

(1) -m=1.n=1.(2) -- m=1.n=2.(3) -- m=2.n=1.



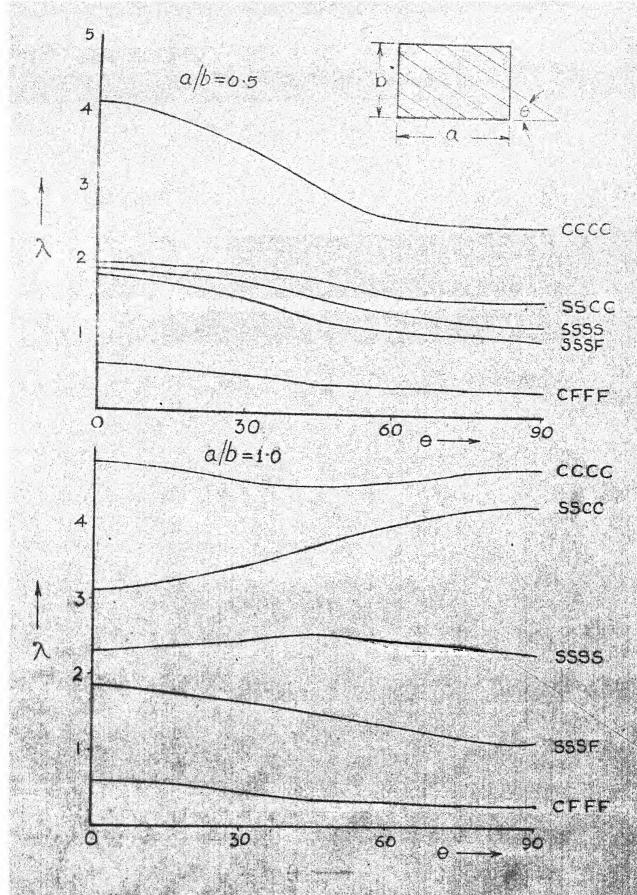


FIG. 5 NATURAL FREQUENCY OF GENERALLY ORTHOTROPIC PLATES WITH VARIOUS B.C.

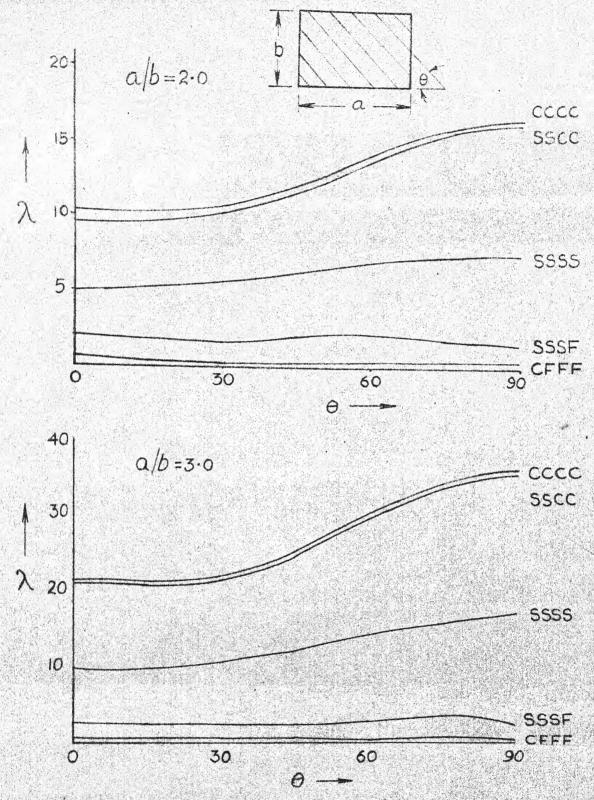
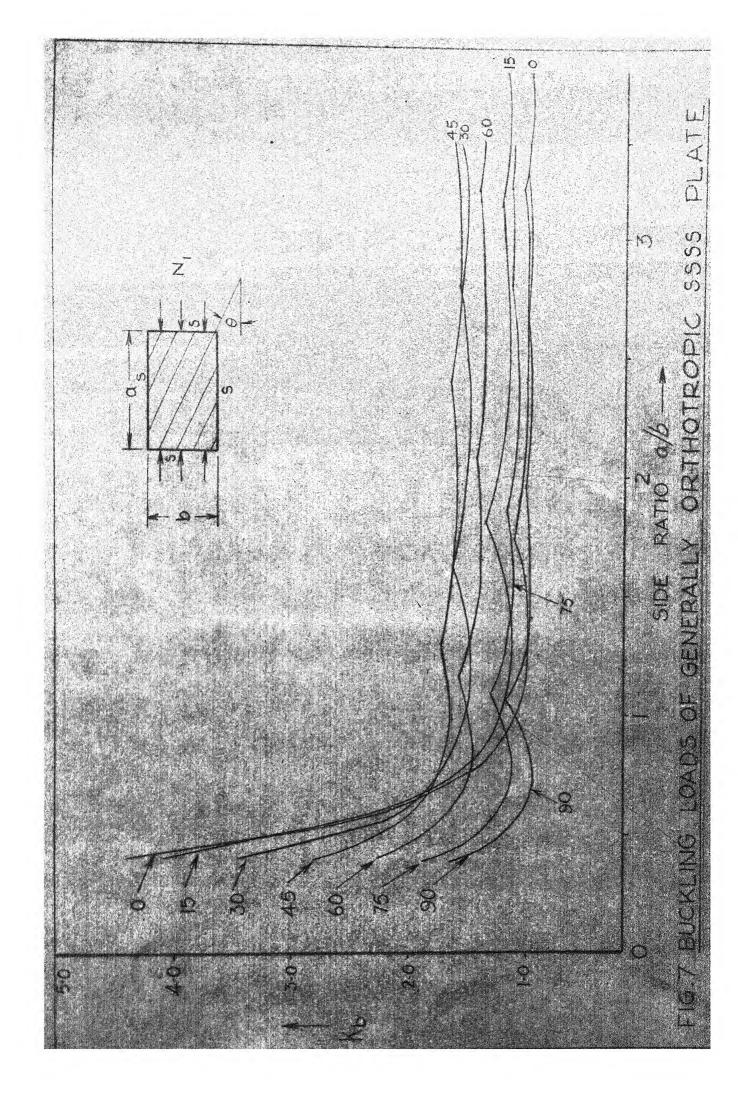


FIG. 6 NATURAL FREQUENCY OF GENERALLY ORTHOTROPIC PLATES WITH VARIOUS B.C.



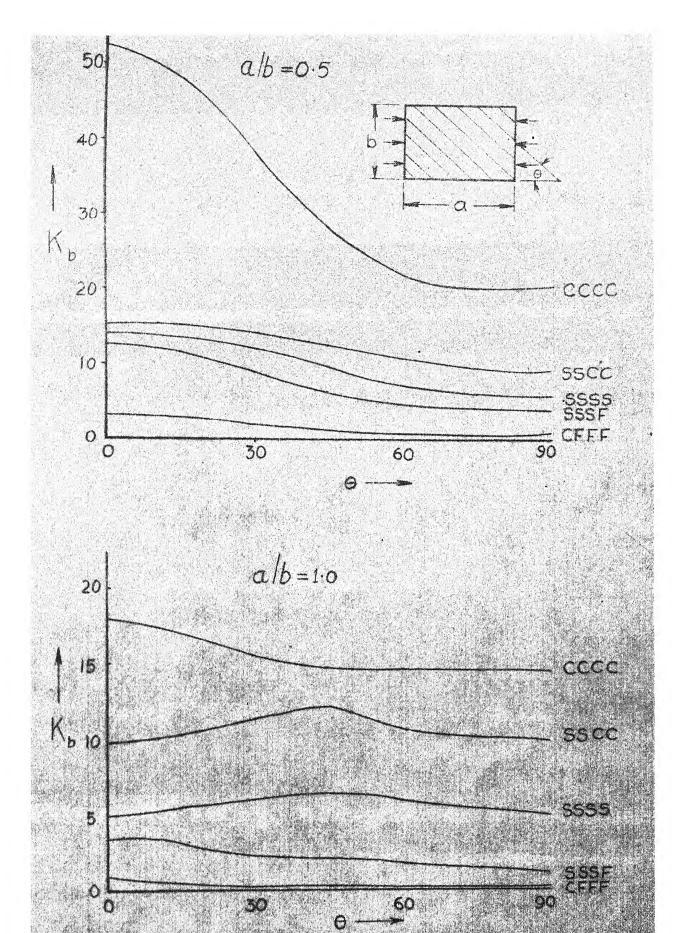


FIG.8 BUCKLING LOADS OF GENERALLY ORTHOTROPIC

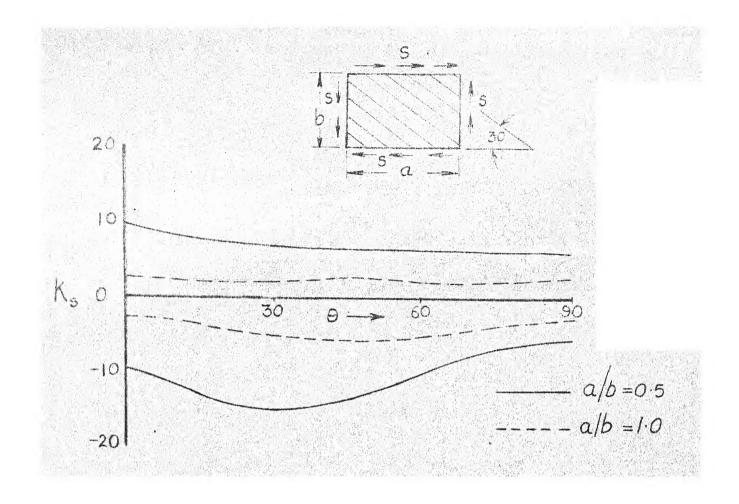


FIG. 9 SHEAR BUCKLING LOADS OF GENERALLY
ORTHOTROPIC SSSS PLATES

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Swarup,
Vibration and buckling of generally orthotropic plates.